ON THE COMPARISON BETWEEN POINT MASS AND COLLOCATION METHODS FOR GEOID DETERMINATION

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ABSTRACT

A comparative study on two methods for geoid determination was performed in the Azores region. In this area several geoid solutions have been computed, mostly with least squares collocation (LSC) in conjunction with the remove-restore technique. The LSC was here substituted by point mass method for the determination of the medium wavelength of the geoid. Both geoid solutions were compared on sea with MSS95 (Mean Sea Surface) and on land with GPS and levelling data.

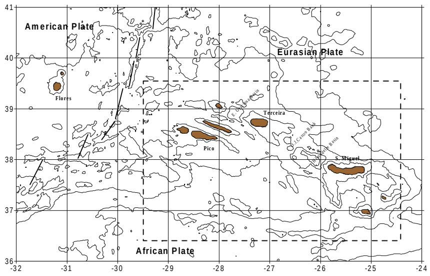

Fig. 1 -Azores Archipelago and surrounding area showing the location of the main tectonic features. The dash line indicate the study area.

1- GRAVITY DATA

The data bank used was: a) ERS altimetric data from 32 days and 168 days cycles; b) EGM96 geopotential model; c) AZDTM98 digital terrain model; and d) gravity anomalies previously validated and obtained from shipboard measurements.
The original data bank, used in previous studies resulted from a compilation of BGI, NGDC and DMA data banks. Most of the data were acquired from USA, United Kingdom and France Institutions in the period from 1970 to 1990. The gravity data was validated by crossover error adjustment fixing the longest track from United Kingdom. This data bank was recently improved with two gravimetric campaigns held in 1998 in the aim of PDIC/C/Mar project (Fernandes *et al.*, 1998) and MARFLUX project (Luis et al., 1999). A new crossover adjustment was done considering PDIC/C/Mar tracks as error free. After adjustment, the standard deviation of the crossover error dropped to 2.61 mGal (from an initial value of 7.21 mGal). In order to fulfil remaining areas with poor coverage, satellite derived gravity anomalies from Andersen and Knudsen (1998) were merged with observed gravity anomalies.

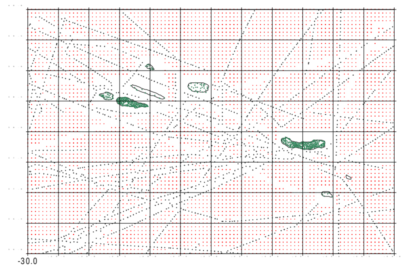

Fig. 2 - Final gravity data distribution including observed gravity anomalies on sea (black dots), on land (green dots) and derived satellite gravity anomalies in areas with absent coverage (red dots).

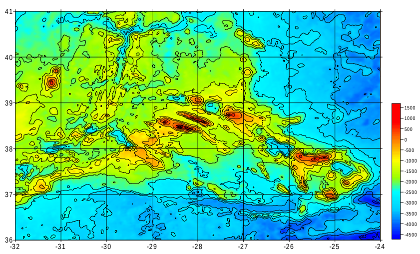

Fig 3 - AZDTM98 bathymetric/altimetric map contoured every 500 m.

2- GRAVIMETRIC GEOID DETERMINATION

The two methods used for the geoid model determination, Least Squares Collocation and Point Mass, were performed in combination with remove-restore technique. The long wavelength component of the gravity anomalies was removed by EGM96 geopotential model, and the terrain effects determined and removed from the residual terrain model AZDTM98.
The point mass, as the alternative method for geoid determination, is also used, in the same way as the other methods, to compute only the medium wavelength component of the geoid.
After the determination of the medium component, the long and short wavelength components of the geoid undulation were restored, respectively, from EGM96 and AZDTM98 models.

Point Mass methodology

It is possible to fit a gravity data set by calculating the gravity effect of a set of point masses ("equivalent sources"), located beneath the gravity data points [Dampney, 1969]. Cordell (1992) proposed an iterative method to determine the point mass set that fits the gravity data, applied specially for interpolation and girding of potential-field data, where the sources are not necessarily masses and are not necessary on a common level compatible with the observation surface. It is the reason why this author calls it as the "generalised equivalent sources".

We have been trying to apply this technique for geoid determination, in substitution of LSC method. The main reason that lead us to this experiment is the simplicity, the low running time of CPU and low memory in computation of the method.

The methodology followed is based on the use of the Newtonian potential function to fit the medium wavelength of the observed gravity anomaly.

$$\Delta g_i(x_i,y_i,z_i)=\sum_{k=1}^{N}\frac{c_k}{\sqrt{(x_i-\xi_k)^2+(y_i-\eta_k)^2+(z_i-\zeta_k)^2}}$$

The set of n constants $\{c_k(\xi_k, \eta_k, \zeta_k)\}$, representing the point masses (sources), and gravity anomaly data $\{\Delta g_i(x_i, y_i, z_i)\}$ refers to a right-handed Cartesian system with z (or ζ) axis down.

This technique is performed in two steps:
1) computation of the n constants (c_k), from the desired gravity anomaly data (Δg_i), using the Cordell's iterative algorithm. This continuos and harmonic functional can be fitted to the data with a mean error of 1 mGal, or higher;

2) computation of the geoid undulation (N), <u>in a bases of planar approximation</u>, with the integral function resulted from the integration of the Δg_i functional in order to the z_i component, multiplied by the factor -1/γ.

$$\Delta g=-\gamma\frac{\partial N}{\partial z} \Leftrightarrow N=-\frac{1}{\gamma}\int\Delta g\,dz$$

$$N_i(x_i,y_i,z_i)=-\frac{1}{\gamma}\sum_{k=1}^{N} c_k \ln\left((z_i-\zeta_k)+\sqrt{(x_i-\xi_k)^2+(y_i-\eta_k)^2+(z_i-\zeta_k)^2}\right)$$

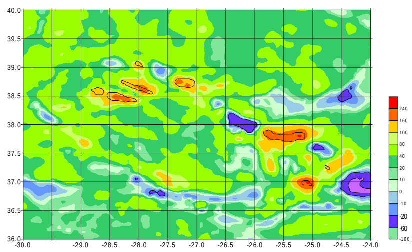

Fig. 4 -Free air gravity anomaly map (contour interval is 20 mGal).

3- RESULTS

From the reduced isostatic gravity anomalies we have applied the iterative method of point mass to determine the set of sources that by the first functional fits the original gravity anomalies.

We have tested different value of depth (ζ_k) for the sources and different number of the iterations. The process was always convergent. The final solution resulted from 12.000 iterations and the sources at 2.500 m below the surface of the gravity anomalies (sea level), resulting a set of 4.461 sources uniformly distributed (due to the field homogeneity).

The residual field (difference between the original and the fitting field) resulted with a standard deviation of 2 mGal. It was possible to reach the 0.5 mGal of standard deviation with 30.000 iterations, but it did not improve the precision on the geoid solution.

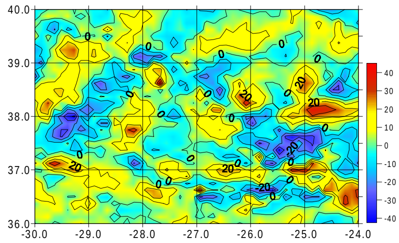

Fig. 5 -Isostatic gravity anomalies (in mGal) fitted by point mass algorithm, with 12.000 iteration and generated by 4.461 sources. The standard deviation of the residuals was 2 mGal.

With the determined set of sources we computed the medium wavelength of the geoid using the second functional and for the same grid as in the collocation solution.

The differences between the two solutions can be computed before or after restoring the geopotential and the residual terrain models, once the models were the same in both solutions.

The statistics of the solutions and of its difference are listed in Table 1. The solutions differ with a standard deviation of only 5 cm.

Both solutions were fitted to MSS95, on sea, and GPS heights on land. We obtained similar results on sea and a small difference on land (7 cm of standard deviation for collocation and 10 cm for point mass method). These statistic results are listed in Table 2.

	Mean	Std	Min	Max
Collocation	55.93	2.62	49.80	60.20
Point Mass	55.93	2.59	49.89	60.22
Collocation -Point Mass	0.0	0.05	-0.19	0.11

Table 1. Statistics of the geoid solutions and its difference (in meters).

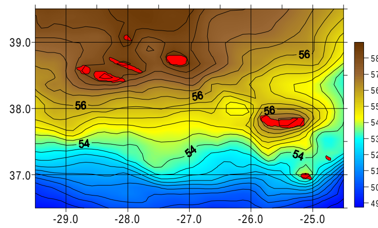

Fig. 6 - Gravimetric geoid model (in meters) around Azores archipelago.

	Original				Fit
	Mean	Std	Min	Max	Std
$Geoid_{Col}$ - $SSH_{(MSS95)}$	-0.09	0.16	-0.47	0.64	0.13
$Geoid_{PM}$ - SSH $_{(MSS95)}$	-0.08	0.16	-0.47	0.52	0.12
$Geoid_{Col}$ -GPS $_{(S.Miguel)}$	-1.01	0.29	-1.38	-0.30	0.07
$Geoid_{PM}$ -GPS $_{(S.Miguel)}$	-0.98	0.29	-1.31	-0.23	0.10

Table 2. Statistics of the differences (in meters) between geoid and SSH from ERS-1, on sea, and GPS heights on land.

4- CONCLUSIONS

Comparing both solutions after fitting them to the SSH and the geoid heights from GPS, one can conclude that they are similar in precision and accuracy.

Considering the consuming time of computation, the necessary space memory, the simplicity and the results of both methods, we conclude that the point mass method is in advantage and is more suitable for local and regional geoid determination from gravity data only.

Theses conclusions leads us to propose the method of point mass as an alternative method for gravimetric geoid determination.

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