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Quantal Aspects in Chemistry and Physics

A tribute to the memory of Professor Couceiro da Costa

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Forward

Some professors left their names intimately associated to the progress of their Schools. Due to their intelligence, exceptional human qualities and devotion to science, as well as deep insight on the role of the latter in future developments, they became reference personalities for future generations. In this context, it is a pleasure to remember Couceiro da Costa, an outstanding professor of Chemistry at the University of Coimbra from 1920 to 1955.

When Couceiro da Costa joined the university, science in Portugal was plagued with many problems that hindered modernization: poor research centers, outdated installations, reduced teaching and research staff, and severe budget constraints. During eighteen years as Head of the Chemistry Department, he did a wonderful job hoping to overcome all such difficulties.

A committee of three Professors of the University of Coimbra and members of the Academia das Ciências de Lisboa, two of them former students of Couceiro da Costa (a chemist, J. S. Redinha, and a physicist, J. da Providência) as well as A. J. C. Varandas, Professor of Theoretical Chemistry, decided to embrace the initiative of organizing a meeting to honor the old master. They imagined that the most significant way of so doing would be by bringing together contemporary reknown scientists to discuss recent progress in a field that was once pioneered by him in Portugal. Entitled "Quantal Aspects in Chemistry and Physics" this meeting is hoped to bring his name before the younger generations, since quantum mechanics and statistical thermodynamics are about the two dearest topics among the ones that he cultivated. Only he could have had then the capacity to follow the development of such matters and be aware of their impact in the scientific preparation of future university generations.

The meeting “Quantal Aspects in Chemistry and Physics” took place at the Academia das Ciências de Lisboa on the 27th of November, 2009. The invited lecturers are from chemistry, physics and mathematics departments of different Portuguese universities and from the Technical University of Denmark. Their lectures are compiled in the present volume.

The success of this initiative has only been possible owing to the goodwill and help of the president of the Academia das Ciências de Lisboa, Prof. Arantes e Oliveira who welcomed this initiative from its incept, the rector of the University of Coimbra, Prof. Seabra Santos for his encouragement, and the vice-Rector Prof. Henrique Madeira for his participation at the Meeting. Thanks are also due to the head of the Department of Chemistry of the University of Coimbra, Prof. Formosinho Simões. The organizing committee is also indebted to the speakers for their valuable contributions. A final word to ackwolegde the editorial care of Dr. Pedro Caridade in the preparation of this volume. The Imprensa da Universidade de Coimbra is also thanked for the publication of the present proceedings.

The Editors

5. The Interpretation of Quantum Mechanics Revisited

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The foundations and meaning of quantum theory became a central issue to Albert Einstein and Niels Bohr since the onset of their impassioned debate in the 1920s, enriched by the contributions of many other distinguished scientists and philosophers. The questions are not settled down at all, despite the great achievements of the theory, its impressive accordance with experiment and predictive power. The fundamental and technological applications range from cosmology to biology, with the development of invaluable instruments and the design of new materials.

Is quantum mechanics a complete or an incomplete theory? Is there an objective reality independent of the observer or is the reality created by the measurements? Are hidden-variable theories justifiable? Is there a quantum theory founded in a local-causal and non-linear approach that formally contains the orthodox linear theory as a special case? Can such a formulation unify classical and quantum physics? Are Heisenberg's uncertainty relations valid in all cases?

Here, the subject is addressed as an adaptation of our contribution to the Colloquium "*Quantal aspects in Chemistry and Physics. A tribute in memory of Professor Ruy Couceiro da Costa*" held at Academia das Ciências de Lisboa, November 27, 2009.

Ruy Couceiro da Costa (1901-1955), University of Coimbra, was one of the first professors and researchers to apply and teach quantum mechanics at Portuguese universities. The above questions presumably crossed his mind as they do pervade, presently, the minds of teachers and researchers interested in the interpretation, philosophy and epistemology of quantum theory.

5.1 Introduction

The roots of quantum theory, originated in the 19^{th} century by Gustav

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Kirchoff's challenge on the black body radiation, were definitely launched between 1900 and 1925. Experiments and first theoretical models on the behaviour of light and material particles recognized that: (*i*) light is emitted or absorbed, and constituted, by photons of energy $E = \hbar\omega$ (Planck - Einstein's relation), where $\hbar$ is the reduced Planck constant and ω the time frequency; (*ii*) atomic energies are quantized, *i.e.*, when an atom "jumps" from an energy level E_i to another E_f it emits or absorbs a photon, $E_f - E_i = \hbar\omega$ (Bohr-Sommerfeld models); (*iii*) to a photon or material particle with momentum, p, is associated a wave number (space frequency) $k = 2\pi/\lambda$, such that $p = \hbar k$ (de Broglie's relation); (*iv*) some atomic emission lines split, in the presence of a magnetic field, into well distinct lines (Zeeman's effect); and (*v*) each electron "orbit" can contain only two electrons (early form of Pauli's exclusion principle underlying the Periodic Table structure).

Classical wave theory and classical mechanics are unable to rationalize the evident wave-particle duality either for light or material particles. The Bohr-Sommerfeld models, though remarkable steps forward, are essentially based on classical mechanics with the introduction of rather *ad hoc* quantum rules, and do not describe most of the atomic and molecular properties. Thus, a well-founded theory was needed to reproduce all known experimental results in such a way that quantum numbers and rules turned out naturally, *i.e.*, without *ad hoc* assumptions.

Since 1925 many quantum formulations were achieved, starting with Heisenberg, Born, Jordan's *matrix mechanics* (1925) and Schrödinger's *wave mechanics* (1926) which, apparently different, were proved to be equivalent by Schrödinger and, independently, by Dirac. However, these formulations are spinless, not fully explaining Zeeman's effect and other spectroscopic details.

Pauli introduced the *spin matrices* (1927) into Schrödinger's time dependent equation, though well-aware that the introduction of spin in such a way was rather *ad hoc*. It is noteworthy that Ralph Kronig firstly, and Uhlenbeck and Goudsmit later, had anticipated the existence of electronic spin in 1925. Curiously, Kronig was much discouraged on the proposal by Pauli who later regretted it.

Between 1928 and 1933, Dirac, based on special relativity, developed a *relativistic quantum theory*, from which spin came out naturally, reproducing Zeeman's effect. Besides, the positron was predicted whose real existence Carl Anderson confirmed experimentally by 1933.

These achievements constitute what is known as the *first quantization*. Yet, an extension to *a second quantization* was still needed to account for the interaction of matter and radiation that Dirac's theory was unable to accommodate. It requires the quantization of fields so that particles turn out as the *quanta of non-classical fields*, allowing the creation and annihilation of quanta in different types of interaction. This is the scope of *quantum field theories*, particularly of *quantum electrodynamics* (Bethe, Tomonaga, Schwinger, Feynman, Dyson, ...) developed since 1948, a very successful theory that reproduces the experimental value of the "g-factor", for example, with an outstanding accuracy.

Further on, *gauge* and *renormalization group theories* (Yang, Mills, Glashow, Salam, Weinberg, Kadanoff, Fisher, Wilson, Gell-Mann, ...) extended quantum field formulations to atomic nuclei (with their "coloured" quarks and gluons), giving birth to *quantum chromodynamics* aiming at the understanding of a myriad of elementary particles and the unification of fundamental forces.

Despite all these great achievements, the gravitation problem still remained. Presently, *superstrings theory* (Green, Schwarz, Witten, ...) appears as the "jewel of the crown" for it attempts the full unification of quantum mechanics and general relativity as well as of the whole physics, what underlies the "dreams of a final theory".

Quantum theories gave rise to a remarkable progress in chemistry and physics, predicting, with great accuracy, the properties of molecules, atoms, nuclei, elementary particles, chemical and nuclear reactions. The mechanisms of the universal forces are unravelled by means of their mediating "particles" and symmetries. The fundamental and technological applications range from cosmology to biology, with the development of invaluable instruments and the design of new materials.

Simultaneously, quantum theory challenges the classical reasoning concerned with causality, determinism, locality and objective reality, shaking the philosophy, ontology and epistemology of science. In this context, since the 1950's, hidden-variable theories were proposed (Bohm, Bell, ...) aiming to recover some of the classical views and experimentally pitting them against the orthodox quantum standpoint.

Recently, the linearity, non-locality and non-causal realm of the orthodox quantum theory has again been questioned. In 2003, José Croca proposed a new approach, based on de Broglie's "pilot wave" idea and radically changing the non-local Fourier ontology of the orthodox quantum theory to a *local wavelet analysis*. A non-linear equation was established and generalized uncertainty relations were derived which, in special cases, lead to Schrödinger's equation and Heisenberg's relations.

5.2 Orthodox Interpretation

Niels Bohr was one of the most brilliant physicists of the 20$^{\text{th}}$ century. The model for the hydrogen atom, the interpretation of quantum theory, the proposal for the uranium enrichment in the 235-isotope and the foundation of the Copenhagen school were, among others, the pillars of Bohr's huge influence in the scientific and philosophical communities all over the world.

As for the theory interpretation, the chief ideas were put forward in the 1920's through the complementary principle and the onset of his debate with Albert Einstein. Bohr, with the contributions of Heisenberg and Pauli, was for certain the precursor of the so-called "Copenhagen or orthodox interpretation" that is, presently, the "standard" for the majority of teachers and researchers [1–4]. It should be noted, however, that Bohr's own ideas and the Copenhagen interpretation are frequently taken as being the same. This is not strictly true. For instance, Bohr avoided the postulate of wave function "collapse" that is central to the Copenhagen interpretation which, in its present form, is essentially based on von Neumann's mathematical formulation [4, 5]. We shall use indistinctly the terms "orthodox" and "Copenhagen" just to convey the standard interpretation and distinguish it from other theories and interpretations such as hidden-variables, many-worlds and non-linear formulations.

Let's then outline the orthodox fundamentals:

- To a free particle with sharp momentum p, and energy E, is associated a *monochromatic harmonic wave* such that $p = \hbar k$ and $E = \hbar\omega$.
- Wave and particle concepts are mutually exclusive, though complementary to rationalize the experimental observations.
- The state $|\Psi\rangle$ of a particle with sharp position, x_0, is described by the Fourier expansion:

$$|\Psi\rangle = \delta(x - x_0) = \hbar^{-1/2}\int_{-\infty}^{+\infty} \exp(-ix_0p_x/\hbar)\exp(ixp_x/\hbar)\mathrm{d}p_x \qquad \Delta x = 0$$

  where $\delta(x-x_0)$ is the Dirac delta function, $\exp(ixp_x/\hbar)$ the eigenfunctions of the momentum operator and Δx the indeterminacy of the position.

In this case, p_x and E are undefined. There exists, however, a set of simultaneous possibilities (the eigenvalues of the momentum operator), each one only really *attributable* through measurements. By means of very many repeated measurements of the momentum, that is, providing that before each measurement the particle is in the same state $\delta(x-x_0)$, a distribution of *different* results is obtained. The momentum indeterminacy, Δp_x, is proportional to the width of the distribution. As the position function encodes all possible momentum values (in this case a continuous spectrum) with equal weights (probabilities) $\Delta p_x = \Delta E = \infty$. Conversely, if $\Delta p_x = \Delta E = 0$, then $\Delta x = \infty$, *i.e.*, the position is undefined. In general:

$$\Delta x \Delta p_x \geq h \qquad \text{(Heisenberg's indeterminacy relation)}$$

This relation means not what is measurable but what is knowable. Position and momentum, for example, are not known simultaneously *before* a measurement on a single particle, *i.e.*, the position or the momentum, or both, are just undefined.

Although Δx and Δp_x are estimated from very many repeated measurements, the indeterminacy principle must not be interpreted supposing that the position and momentum of a single particle are defined simultaneously before a measurement, and that the principle expresses the uncertainties of statistical errors due to observation disturbances and instrumental incompatibilities.

Such errors are generally present, but one can, at least conceptually, eliminate them. Even so, an ideal errorless measurement of the position on a single particle in any state $|\Psi\rangle$ would generate, non-deterministically, an eigenstate $\delta(x - x_0)$ which is Fourier composed by an infinite number of momentum eigenstates. If this is followed by an errorless measurement of the momentum it generates, non-deterministically, an eigenstate $\delta(p_x - p_0)$ with sharp momentum, p_0, which is Fourier composed by an infinite number of position eigenstates, and so forth. The same scenario results for other properties.

In the context of Fourier analysis, the indeterminacy is intrinsic, not a question of statistical errors. The reality of physical properties, that is, the existence of effective values for them, depends on the measurements. Yet, there is an *empirical reality* which not being independent of measurements *leads to the same predictions for all observers*.

Associating a monochromatic harmonic wave to a material particle with sharp momentum has, however, a physical inconsistency. In fact, the wave phase velocity, v_{pha}, and the particle velocity, v_{par}, are related by $v_{\text{pha}} = c^2/v_{\text{par}}$, where c is the light velocity. Therefore, the wave will precede the particle since $v_{\text{par}} \ll c$. But one can suppose that to the particle is associated not a monochromatic wave but a wave packet:

$$\Psi(x,t) = \int_{-\infty}^{+\infty} f(x)\exp[i(kx - \omega t)]\mathrm{d}k$$

with k within a narrow interval $k_0 \pm \Delta k$. Then, the group velocity is:

$$v_g = \left(\frac{\partial\omega(k)}{\partial k}\right)_{k_0} = v_{\text{par}}$$

and

$$p = mv_g = \hbar k_0; \quad E = \hbar\omega_0$$

Apparently, this resolves the problem. Besides, if a Dirac delta function is ascribed to a particle with sharp position, the momentum and the energy are totally undefined. The wave packet also circumvents this point. Then, a classical image comes out: a particle *more or less* localized moving in space-time, and encoding de Broglie's relation and Heisenberg's indeterminacy principle.

Another physical inconsistency, however, turns out: material wave-packets disperse rapidly so any image of a trajectory is nonsense. But, what about the apparent trajectories of particles observed in cloud chambers? This question motivated Heisenberg to set out his indeterminacy relations [1, 2, 4].

Obviously, there are some uncomfortable physical details in the above analysis and assumptions, at least against common sense. Yet, one thing is absolutely certain: the orthodox quantum mechanics gives results in an excellent agreement with experiment. A philosophical standpoint is then inescapable.

5.3 Philosophy and mathematics

Harald Høffding was Bohr's teacher of philosophy and a close family's friend. He is reported as having had a considerable influence on Bohr's philosophical standpoints. Høffding defended that in our endeavour to get knowledge there exists an irreducible irrational residue impossible to overpass whichever our efforts are. It appears that Bohr agreed on this view by saying that "such a residue is, in quantum mechanics, mathematically expressed in a lucid form". He also asserted: "There is no quantum world. There is only an abstract physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature" [4, 6].

This way of thinking follows, in some ways, the philosophical position of *positivism*. It claims that for a scientific statement being meaningful it has just to be a formally logical and verifiable statement. The objective reality (a reality flowing "out there" independent of the observer-instruments) is, for positivism, a metaphysical concept since it is not possible to know of a universe without observing it. If one has two theories *formally* logical and experimentally verifiable, then both theories are valid in principle. The choice between them is generally a matter of convenience or simplicity, independently of their assumptions might be *physically* contradictory or unrealistic. The matrix and wave formulations of Heisenberg and Schrödinger are typical examples. In a strict sense, one can think of abstract concepts (physical properties) to describe the systems but for positivism they only become real upon observation or measurement. Such physical properties are a kind of "dummy variables", undefined entities, until one can attribute to them a quality or a quantity by observation or measurement. Otherwise, they are abstract objects not existing in reality.

The point of view of *realism*, defended by Einstein, is different (in part only, we think) of the one of positivism. For a realist, logic and measurement are certainly essential ingredients of science. But, according to Einstein, for a physical property to exist in reality it suffices that: "if, without in any way disturbing a system, we can predict with certainty (*i.e.*, with a probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity" [7]. For a theory to be considered complete there must be a one-to-one correspondence between the elements of physical reality and the elements of the theory (its physical concepts). Otherwise, the theory is incomplete. Einstein asserted: "Physics is an attempt to capture the reality as it is thought to be, independently of being observed or not" [6].

The mathematics underlying the orthodox theory is, ultimately, Fourier analysis (just one of the possible mathematical techniques to represent functions) that is endowed with a full physical meaning as an ontological principle.

In the orthodox view, quantum mechanics is a complete theory for it is formally logical and vindicated by experiment. As such, it is frequently considered as the "end of the road" of our possible knowledge. It is suggested, however, that Bohr himself was not a radical positivist but to some extent a *pragmatic*. Quoting Jim Baggott: "The pragmatist doctrine admits a more practical (or, indeed, pragmatic) approach to the reality of entities — such as electrons — whose properties and behaviour are described by theories and which produce secondary observable effects but which themselves cannot be seen. According to the pragmatist, what we can know is limited not by we can see but by we can *do*. It seems logical that the father of the modern atomic theory would want to accept the reality of atoms. But Bohr placed limits on what a theory of the internal structure of the atom could say. He argued that we live in a classical world, and our experiments are classical experiments. Go beyond these concepts, and you cross the threshold between what you can know and what you cannot. Positivist or pragmatist, the most important feature of Bohr's philosophy is that he was anti-realist. It denied that quantum theory has anything meaningful to say about an underlying physical reality that exists independently of our measuring devices. It denied the possibility that further development of the theory could take us closer to some yet unrevealed truth" [4].

We do not believe in "end-roads" in science. Nevertheless, we do acknowledge that once a consistent set of axioms and rules is established it has always an "end-road". Also, it is not always possible, within such a set, to decide if some mathematical propositions are true or false, as Kurt Gödel demonstrated. Yet, this does not mean that other structures extending the roads of science are precluded.

John von Neumann's formulation of the quantum theory is an unassailable mathematical structure, based on a consistent set of postulates and rules. However, von Neumann asserted: "In spite of the fact that quantum mechanics agrees well with experiment, and that it has opened up for us a qualitatively new side of the world, one can never say of the theory that it has been proved by experience, but only that it is the best known summarization of experience" [6, 8]. It is noteworthy that von Neumann (who never was a fellow of the Copenhagen school) did not adhere to all Bohr's views. For example, his interpretation on the nature of measuring devices was different from the one of Bohr [4].

Whatever the discussion is, it seems to us that, after all, most of the scientists, if not all, adhere to positivist-pragmatist methodologies, though with realist outlooks. Indeed, who are the researchers that (nowadays, at least!) submit a project stating that the objects of their proposed investigations *only become real upon measurement?*

5.4 Orthodox theory

In 1932, John von Neumann established the rigorous mathematical foundation of the orthodox theory in the context of Hilbert's vector space [8], also explored by Dirac [9]. It is commonly expressed by a set of postulates the most important of which, for the present discussion, are:

a) To each physical state of an individual system, at time t, corresponds a normalized vector of Hilbert's space, $|\Psi\rangle$, which describes, *completely*, the system.

b) To a physical observable, A, corresponds in Hilbert's space a linear Hermitian operator, $\hat{\mathbf{A}}$, which has a complete and orthonormal set of eigenvectors (a *basis*), $|\alpha_i\rangle$, and corresponding eigenvalues, A_i, such that:

$$\hat{\mathbf{A}}|\alpha_i\rangle = A_i|\alpha_i\rangle \qquad (i = 1, 2, \ldots)$$

were the A_i's are the only possible values obtainable from any measurement of A.

c) If A is measured on a general state $|\Psi\rangle$, the strongest predictive statement that can be made is that the probability of obtaining the value A_k is: $|\langle\alpha_k|\Psi\rangle|^2$.

d) A measurement generally changes non-deterministically the state vector. Regardless of the state before the measurement, immediately after it the new state will coincide with the eigenvector corresponding to the obtained eigenvalue (this is the so-called *reduction* or *collapse* of the state vector).

From the postulates, it is straightforward to prove the *expansion theorem*, *i.e.*, a state vector $|\Psi\rangle$ can be expanded into the vectors of any basis:

$$|\Psi\rangle = \sum_i \langle\alpha_i|\Psi\rangle|\alpha_i\rangle$$

The Fourier composition of Dirac's delta function, seen above for a particle with sharp position, is just a particular case of the expansion theorem.

The postulates clearly mean that the direct link between cause and effect is severed. This is the big clash with classical mechanics. Indeed, it is asserted that measurements on *exactly* the same state can give different results, *i.e.*, an initial state does not uniquely determine future outcomes. Only if the state vector coincides with an eigenvector of an operator $\hat{\mathbf{A}}$, for example $|\alpha_k\rangle$ can one be certain that repeated measurements of the observable A give always the same result, A_k. However, in such state, repeated measurements of another observable B, whose operator, $\hat{\mathbf{B}}$, does not commute with $\hat{\mathbf{A}}$, can give different results. Moreover, a measurement in the context of the postulates, which capture the Copenhagen interpretation, remains an unexplained process, since there is nothing in the mathematics that specifies how and when the wave function collapses.

From the postulates, it is also straightforward to prove the *compatibility theorem*:

> *"Given two observables A and B with corresponding operators,* $\hat{\mathbf{A}}$ *and* $\hat{\mathbf{B}}$ *, any one of the following conditions implies the other two: (i) A and B are compatible observables; (ii)* $\hat{\mathbf{A}}$ *and* $\hat{\mathbf{B}}$ *have a common eigenbasis; (iii)* $\hat{\mathbf{A}}$ *and* $\hat{\mathbf{B}}$ *commute".*

In text books, it is not often noted that the theorem does not assert the impossibility of two non-commuting operators having *some* eigenvectors in common, but just the impossibility of *all* the eigenvectors of a basis being common. For example, the x and z operators of the angular momentum do not commute but have *some* eigenvectors in common [10].

Bohr and Einstein discussed this matter privately. Quoting Ballentine:

> "… quantum mechanics, properly understood, does not prohibit or restrict simultaneous measurement of non-commuting observables, but rather it does not deal with such measurements at all" [11].

In fact, according to Bohr, a unique instrument for simultaneously measuring incompatible observables is not conceivable. Two different devices are needed to measure such observables of the system in a given state, being the respective results always limited by the indeterminacy relations.

5.5 Time-dependence. Many-worlds formulations

According to the orthodox theory the state vector $|\Psi\rangle$ or/and the operators can evolve in time through two distinct processes: (*i*) perturbing the system by measurements, leading to non-deterministic results; or (*ii*) letting the system unperturbed.

In the last case, the evolution is deterministic, obeying to motion equations. These depend on the particular formulation one adopts. By 1930, three different, but equivalent, pictures were definitely settled down by Heisenberg (matrix mechanics), Schrödinger (wave mechanics) and Dirac (interaction picture) [9,12], each with its own importance to further developments. Here, the non-relativistic Schrödinger's equation is adopted:

$$\hat{\mathbf{H}}|\Psi\rangle = i\hbar\frac{\partial|\Psi\rangle}{\partial t}$$

where $\hat{\mathbf{H}}$ is the Hamiltonian operator.

Schrödinger established the equation from the concept of wave packets for free-particles and assumed its validity for all cases [12]. Thus, it commonly constitutes a further postulate of the theory.

Schrödinger was a realist believing that the wave functions exist in reality as amplitudes of a "material field scalar". He interpreted the wave-particle duality in pure undulatory terms assuming that the transitions between standing waves, that describe the stationary quantum states, are smooth and continuous. In this way he hoped to explain the apparent non-classical atomic properties with essentially classical concepts, restoring the determinism and causality that the theory appeared to abandon. He viewed an electron as a superposition of wave disturbances (wave packet) resulting in its particle-like properties.

Schrödinger's interpretation clashed with Heisenberg's matrix mechanics. Initially, Heisenberg considered the electron essentially as a corpuscle-like entity, supposedly with defined positions and momentum, and subjected to discontinuous jumps between stationary states. Matrix mechanics was, to him, no more than an operational algorithm and the uncertainty principle expressed instrumental disturbances and incompatibilities, turning the simultaneous specification of the position and momentum impossible. The heated rivalry between the two young men was tempered by Bohr and Pauli. Heisenberg soon did adhere to Bohr's interpretation based on the complementary principle. Schrödinger never did [2–4, 6, 13, 14].

Hendrik Lorentz pointed out to Schrödinger the rapid spreading of the material wave packets dispersing into wider amplitude distributions [4]. Besides, the wave functions derived from the equation are generally complex and multi-dimensional which does not seem compatible with the realist interpretation that Schrödinger pretended. Also, any function $|\Psi\rangle\exp(i\o)$, where $\o$ is an arbitrary phase factor not experimentally accessible, is a valid solution to the equation.

These issues were conveyed, in 1926, by the humouristic ditty [6]:

Erwin with his "psi" can do Calculations quite a few.
But one thing has not been seen:
Just what does "psi" really mean?

The difficulties were circumvented, also in 1926, by Max Born's interpretation [1]: the wave function is an abstract non-local entity, just giving the probability density, $|\Psi|^2$. However, this raises another question: how can *abstract* entities explain the diffraction and interference observed in the two-slit experiment?

Furthermore, the collapse of the wave function, considered by some authors [15] as a "recipe" rather than an axiom, gave rise to an intricate and puzzling question. In fact, according to the orthodox view, the measuring device is not independent of the observed system, constituting with it an isolated super-system. If one includes this super-system in the deterministic Schrödinger's equation, a succession of entangled states is always obtained along the time, with no collapse of the total wave function in order to select one of the possible eigenstates. If another observer of the super-system is introduced, the argument repeats with no way out. This is the heart of the famous "Schrödinger's cat paradox" [4, 6].

For cosmology, at least, a question is inescapable: how was the universe created? According to the above, the collapse of the universe super-wave function is not possible, unless we postulate it. Therefore, only a virtual bunch of simultaneous possibilities could exist. The *many-worlds interpretation*, put forward in 1957 by Hugh Everett III, assumes the deterministic side of the orthodox theory, but dismisses the postulate of the wave function collapse. It asserts the real existence of all possibilities (*worlds*) with a crucial detail: we, ourselves, are only conscious of just one of such worlds. This interesting interpretation has suffered alterations and refinements (for instance, the *many alternative histories of the universe*) that are taken seriously by many researchers, particularly cosmologists. Indeed, *quantum cosmology* can not resort to repeated measurements of the universe like the common ones in physics and chemistry laboratories. Furthermore, other approaches have been proposed either avoiding the collapse of the wave functions or introducing additional terms to Schrödinger equation to cause the collapse [4, 15–17].

5.6 Einstein, Podolsky and Rosen thought experiment (EPR)

The debate between Einstein and Bohr culminated at the 5th and 6th Solvay Conferences, held in Brussels in 1927 and 1930. Einstein asserted the incompleteness and inconsistency of the orthodox quantum theory through a series of thought experiments suggesting, on the one hand, that the theory implied a weird instantaneous action at a distance and, on the other hand, that the position-momentum and energy-time uncertainty relations could be violated. Bohr was able to rebut Einstein's arguments. Ironically, the famous "photon box experiment" was brilliantly dismantled by means of Einstein's general relativity theory.

The debate recommenced in 1935, when Einstein, Podolsky and Rosen (EPR) published a paper, entitled "Can quantum-mechanical description of physical reality be considered complete?" [7]. In the words of Léon Rosenfelf, who was at that time in Copenhagen: "… this onslaught came down upon us as a bolt from the blue" [6].

The EPR arguments can be outlined as follows. Consider two particles, A and B, initially interacting and moving apart. Suppose they reach a relative distance of years-light where they should be separate entities with independent reality, *i.e.*, there is no longer any interaction between them (*the separability* *assumption*). Additionally, EPR adopted a criterion for reality: "if, without in any way disturbing a system, we can predict with certainty (*i.e.*, with a probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity".

They also accepted that position (q) and momentum (p) of each individual particle cannot be known simultaneously according to Heisenberg's indeterminacy principle. Yet, both $q_A - q_B$ and $q_A + q_B$ can be sharply defined since the respective operators commute.

Now, if one measures q_A, then q_B is predicted (from $q_A - q_B$) without perturbing B. Therefore, the position of B must have a definite value, according to the above criterion, even if A is not measured (in this case, of course, we would not know what it was, but this not affect the argument). In other words, a measurement of A only affects *our knowledge* of the properties of B which were already defined before the measurement.

If one measures p_A, instead of q_A , then p_B is exactly predicted (from $p_A + p_B$) without perturbing B. So, its momentum is also an element of reality. Thus, the position and momentum of B must be, simultaneously, elements of physical reality, *i.e.*, well-defined independently of any measurement, in contradiction with the orthodox theory. Otherwise, the reality of q_B or p_B would depend upon the choice of the measurement on A, whose disturbance would be instantaneously felt by B, years-light apart from A. This would imply a "spooky" action at a distance, violating *locality* and against special relativity. EPR claimed: "No reasonable definition of reality could be expected to permit this".

The paper ends with the statement: "While we have thus shown that the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible."

It should be noted that EPR did not intend, contrary to other Einstein's thought experiments, to challenge Heisenberg's indeterminacy relations. It was not designed to simultaneously measure the position and momentum of a particle but just to demonstrate the *observer-independent* reality of both. The "onslaught" was against the inseparability and non locality implicit in the orthodox theory.

Bohr's answer [18] disagreed on the EPR criterion for physical reality, though conceding that there was no "mechanical" disturbance of particle B due to a measurement on particle A. The role of the measuring device was emphasized in the sense that any quantum object and the measuring device constitute an indivisible whole (the "phenomenon"), there being no room for a physical disturbance due to an observation.

Bohr's wording was not much clear, as he later admitted, but its essence seems to be the following. The wave functions of the two-particle system are *inseparable*, *i.e.*, they are global instances of the same entity. The particles behave as they were just one at all distances. Once they have interacted, they are entangled for ever. The wave functions are Fourier composed by the same basic elements: monochromatic harmonic waves extending through all space and time, affected by the proper coefficients.

If one measures q_{A}, then q_{B} is predicted from $q_{\mathrm{A}} - q_{\mathrm{B}}$ but, according to the indeterminacy relation, p_{A} is unknown and so is p_{B}, even knowing $p_{\mathrm{A}} + p_{\mathrm{B}}$, as also admitted by EPR. The position or the momentum of any particle only can be attributed by means of observations, requiring two distinct and incompatible devices (two different "phenomena") that preclude the simultaneous definition of the position and momentum. Thus, due to the inseparability of the entwined particles and the measuring device subjecting A to a position observation (one "phenomenon"), for example, is practically the same as observing B conferring to it a well-defined position. But the sharp definition of the momentum of B is a different "phenomenon".

Although Bohr agreed that there was no "mechanical" disturbance of B due to an observation of A, he maintained that a measurement of particle A somehow instantaneously "influences" particle B. He did not explain this "influence on the very conditions which define the possible types of predictions regarding the further behaviour of the system", and concludes that since "these conditions constitute an inherent element of the description of any phenomenon to which the term ‘physical reality’ can be properly attached, we see that the argumentation of the mentioned authors does not justify their conclusion that quantum-mechanical description is essentially incomplete”.

Will the so-called “spooky” action at a distance (where something that happens to a particle at a location can instantaneously be reflected in other particle at a huge distance) violate the limit of light velocity? In a strict orthodox interpretation it seems that the relativity theory is not at stake. As said above, the two particles, though very far apart, continue to belong to the same physical entity. *Only* observing one of them its attributes become defined, which *correlates* to what would be observed in the other particle. But no information or signal is transmitted, at least in conventional terms, and there is no traditional relation of cause-effect. Actually, it is shown that quantum entanglement cannot be used to instantaneously send conventional and useful information. Quoting Brian Green: “the special relativity theory survives by a *hair’s breadth*” [19].

It appears that, within the mathematical framework of the non-local Fourier analysis, which is the basis of the orthodox theory, Bohr’s answer to EPR is right, suggesting that the limit of what can be known has been reached. However, it lays against the heart of Einstein’s realist belief in physical separability and locality which, once assumed, certainly support the EPR conclusions. This is the very realm of the debate. Apparently, Einstein did not question the linearity and Fourier analysis underlying the orthodox theory. Nonetheless, the EPR thought experiment paved the way to other theories, interpretations and real experiments that challenge the orthodox “end of the road”.

5.7 Hidden-variable theories

The randomness implicit in the orthodox postulates means that repeated measurements under exactly the same initial conditions can give different results. The initial wave functions only allow probabilistic predictions concerning the outcome of future measurements.

The main objective of hidden-variable theories is to remove such randomness, assuming that initial states, apparently identical, are really different and distinguishable by variables not specified ("hidden" variables). Therefore, the states defined in the orthodox theory by the wave functions would not correspond to precise values of these variables, but to averages over them. If it is possible to set up other states specifying the precise values of such variables, then the classical causality will be restored with no need of appealing to the unexplained wave function collapse. Quoting David Bohm: "lawlessness of individual behaviour in the context of a given statistical law is, in general, consistent with the notion of more detailed individual laws applying in a broader context" [20]. This presupposes the existence of a deeper quantum-mechanical level that explains the statistical realm of the orthodox theory, similar to Brownian motion theory and statistical mechanics.

The idea was lurking since Einstein's work on spontaneous and stimulated emission of radiation by molecules (1916-1917). By that time he manifested to Max Born his discomfort about the fact that quantum theory could not predict the time and direction of the photons emission, letting the details to chance and renouncing complete causality. By 1927, he even attempted a kind of hidden variables formulation, introducing a "guiding field" to real particles, and submitted a paper that he soon withdrew for having dismissed the idea. Nonetheless, it seems to have influenced, in some way, Born's interpretation and de Broglie's "double-solution" based on the "pilot wave" suggestion [4].

In 1932 von Neumann stated the "impossibility theorem" [4, 8], apparently proving that "no hidden-variables theory can reproduce and explain all the results of the orthodox theory", which certainly discouraged the pursuit of hidden-variables for the next twenty years. In fact, David Bohm revived it by 1952, impressed by the EPR experiment, and based on the older proposal of de Broglie's "pilot wave" according to which the wave function is a guide to the motion of the particle that likely follows the path where the wave intensity is larger [4,6,21,22].

Briefly, in the so-called de Broglie-Bohm hidden-variables theory, a system, at each instant, is described by a wave function (solution of Schrödinger's equation) and by the positions and momenta of all the particles. A "quantum force" is calculated from the wave function and added to other forces (coulombic, van der

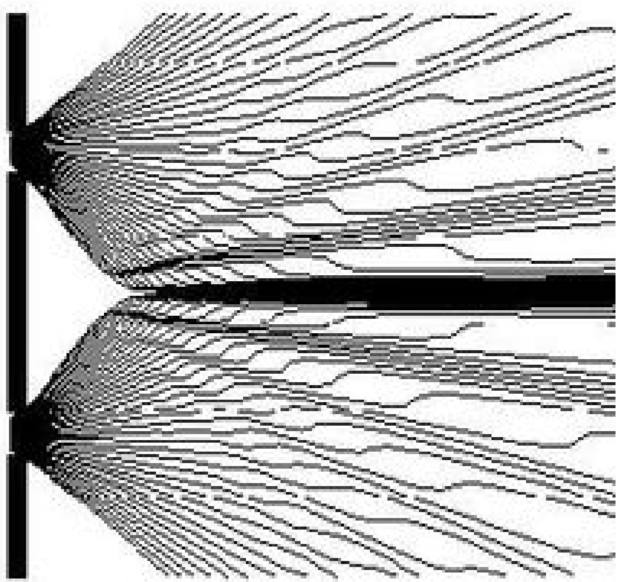

Figure 5.1. Two-slit results from de Broglie-Bohm hidden variables theory; adapted from [23].

Waals, etc.). The trajectories of the particles are then calculated by integrating Newton's motion equations. Figure 5.1 shows the theoretical results for the two-slit experiment. Apart the *sui generis* trajectories, the main conclusions are: (*i*) agreement with the orthodox theory; (*ii*) exact particle paths; and (*iii*) the "quantum force" operates instantaneously over arbitrarily large distances accounting for diffraction and interference effects.

The last conclusion clearly implies the violation of locality, one of Einstein's sacred beliefs. Nonetheless, the results contradict von Neumann's impossibility theorem. As matter of fact, von Neumann's proof is mathematically impeccable, but one of the primary assumptions, concerned with certain observable averages, is physically restrictive, turning the theorem incorrect as conjectured by de Broglie and Bohm, and proved by John Bell [24, 25]. Besides, inspired by Bohm and EPR, Bell derived a mathematical inequality for correlated properties (spins, for example) of two interacting particles (photons or electrons) assuming: (*i*) an objective reality; and (*ii*) the preservation of locality, according to the

EPR claims. Bell's inequality, $-2 < C(A, B) < +2$, where $C(A, B)$ are correlation coefficients, can be tested by real experiments. If the inequality is verified then the conclusions of EPR are experimentally confirmed and *local* hidden-variables theories justified.

During the 1970's and 1980's a series of experiments were carried out by the teams of John Clauser and Alain Aspect [4, 6, 16]. The majority of the results pointed to the violation of the inequality. However they did not appear conclusive on whether both Bell's assumptions should be dropped or just that of locality, *i.e.*, the experiments opened the possibility of a non-local objective reality. More recently, analyses about "loopholes" and bias on Bell's inequality tests have been reported [26–28].

In 2006, the Austrian-Polish group led by Markus Aspelmeyer and Anton Zeilinger tested a new inequality derived by Anthony Leggett [29] who altered Bell's inequality by assuming instantaneous influences through entangled particles and pitting non-local hidden variable theories against the orthodox quantum mechanics. The experimental results [30] point to the violation of Leggett's inequality according to the predictions of quantum mechanics. Soon after, Branciard *et al.* [31] claimed that the falsification of the inequality was flawed and proposed new inequalities to test Leggett's model, though the results also point to the agreement with the orthodox theory. Yet, Alain Aspect asserted that the violation of Leggett's inequality implies only that realism and a certain type of non-locality are incompatible, that is, it does no rule out *all* possible non-local models [6]. So, the matter is not settled down at all.

It is worth mentioning that Einstein became very interested in Bohm's ideas but soon disliked hidden-variable theories considering them "too cheap". To him, no amendments to the orthodox theory should be made. He accepted quantum mechanics as a correct statistical theory, though the wave function only described the behaviour of an ensemble of systems, not an individual system. Thus, he sought, without success, a new deeper *unified field theory* from which the orthodox statistical realm could come out naturally and meaningfully.

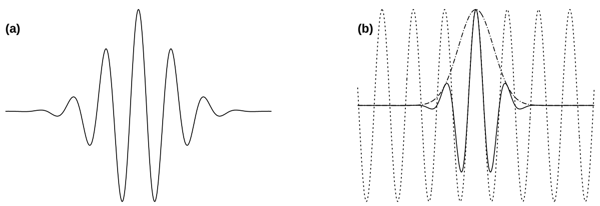

Figure 5.2. (a) Morlet's wavelet; (b) Gaussian modulation of a monochromatic harmonic wave.

5.8 A local-causal and non-linear approach

Recently a new approach to quantum physics has been put forward by José Croca and collaborators [32,33]. It should be said at the outset that the approach does not intend, in any way, to amend the orthodox theory. On the contrary, it radically changes the ontology by adopting local wavelets instead of non-local Fourier analysis. The fundamental assumptions are:

(i) The existence of an objective reality, causal, local and non-linear. Particles have defined positions and momenta even in the absence of measurements.

(ii) Local wavelet analysis instead of non-local Fourier analysis.

Wavelets are, essentially, wave entities localized in time and frequency, contrasting to monochromatic harmonic waves that are localized in frequency but extend infinitely in time and space [34]. The main argument is: real wave signals are always localized in space-time, generally with well-defined frequencies, whereas the infinite monochromatic harmonic waves (underlying Fourier compositions and the orthodox quantum mechanics) are ideal entities devoid of physical reality.

For example, the basic Morlet's wavelet:

$$\Psi(x,t) = \exp\left[-\frac{(x-vt)^2}{2\sigma^2} + i(kx-\omega t)\right]$$

is just a monochromatic harmonic wave modulated by a Gaussian function of width σ, *i.e.*, a localized entity (see Figure 5.2) that encodes a well-defined frequency. Note that when $\sigma \to \infty$ the harmonic wave is recovered. Taking wavelets as the building blocks for composing functions it is pos-

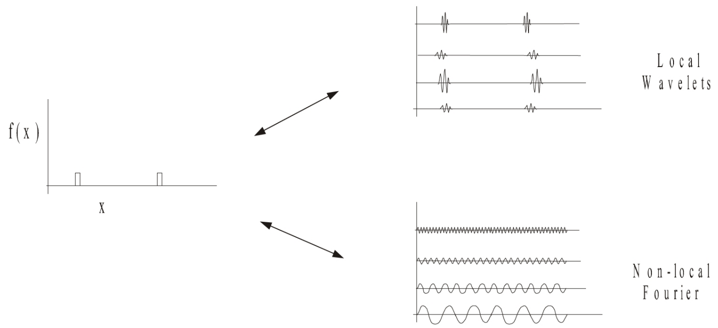

Figure 5.3. Local wavelet versus non-local Fourier analyses [32].

sible to carry out a local analysis of a given function, instead of a non-local Fourier composition for which the building blocks are monochromatic harmonics completely delocalized in space-time. The physical meaning of this can be grasped from Figure 5.3.

Suppose that $f(x)$ represents, at a given instant, the positions of two particles in a relative motion. If the position of one peak moves, it is only necessary to handle the respective group of wavelets to recompose it, *independently* of the position of the other peak that is composed by another group of wavelets. Yet, Fourier analysis takes $f(x)$ as a whole, composed by monochromatic harmonic waves extending through all space-time. Thus, the motion of one peak implies the global reconstruction of $f(x)$ by the *same* harmonic waves altering, of course, the respective coefficients. The last analysis clearly implies the entanglement of the particles as asserted by the orthodox theory, even if they are years-light apart. Both mathematical analyses are unassailable though leading to distinct physical pictures: locality versus non-locality.

(*iii*) A basic natural chaotic sub-quantum medium where all physical processes occur. Particles are complex entities, stable organizations of the sub-quantum medium, composed by a guiding wave (θ), responsible for the interferometric properties, enclosing a very narrow localized structure (dubbed as "singularity" or "acron", ξ), related to the particle size and re-

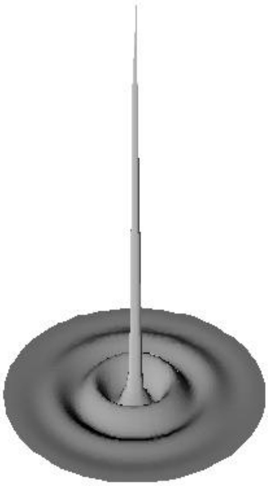

Figure 5.4. Sketch of a quantum particle [32].

sponsible for the usual quadratic detection. This is sketched in Figure 5.4. The acron carries most of the energy. The theta wave, with practically no energy, guides the acron by a non-linear interacting process preferentially to regions where the intensity is higher. The non-linear process implies that the two components of the entity beat always in phase. The orthodox *indeterminacies* should now be interpreted as the ever-present statistical *uncertainties* in the measurement processes. This is, essentially, a revival of de Broglie's "pilot wave" suggestion.

(*iv*) A non-linear, non-relativistic and time-dependent master equation, that combines the corpuscular and wave sides of classical physics, established from the Hamilton-Jacobi and fluid continuity equations:

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + \frac{\hbar^2}{2m}\,\frac{\nabla^2(\Psi\Psi^\star)^{1/2}}{(\Psi\Psi^\star)^{1/2}} + V\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

For the special cases of potential $V = 0$ and stationary solutions, the master equation is formally identical to Schrödinger's equation. On the other hand, when the corpuscular and undulatory properties are taken as independent realities, the master equation leads to the fundamental classical equations. Thus, it is claimed that the approach unifies quantum and classical physics.

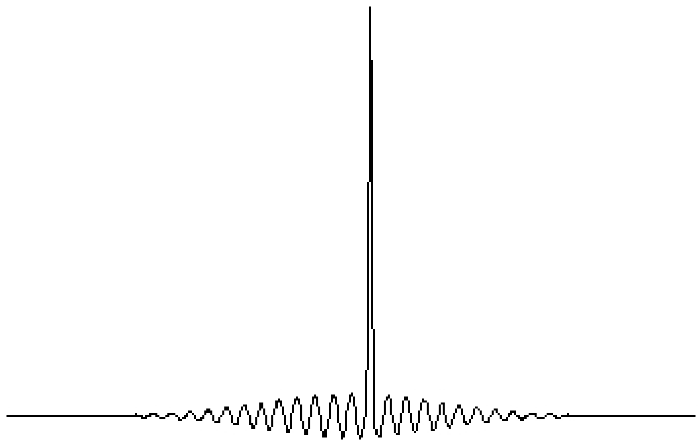

Figure 5.5. Free particle model [32].

The model analytical solution of the master equation for a free particle, for example, is:

$$\phi = \xi + \theta = \sqrt{\frac{E}{\pi^{1/2}\sigma_0}}\left\{\exp\left[-\frac{1}{\hbar^2}\frac{(xp_x - 2Et - \varepsilon_0)^2}{2\sigma_0^2}\right] + \alpha \exp\left[-\frac{1}{\hbar^2}\frac{(xp_x - 2Et - \varepsilon)^2}{2\sigma^2}\right]\right\}\exp\left[\frac{i}{\hbar}(xp_x - 2Et)\right]$$

where σ_0 and σ are, respectively, the widths of the acron (very narrow) and the θ wave; ε_0 and ε are translation parameters for the acron and θ wave, so that the acron is always inside the guiding wave, and $0 < \alpha \lll 1$; $E = \hbar\omega$ and $p = \hbar k$. The real part of the function is represented in Figure 5.5, which is captured by the sketch of Figure 5.4.
According to this approach, the quantum particle is interpreted as a wave pulse, with defined energy and frequencies, described by the non-linear equation and moving without dispersion. This resembles the soliton phenomena [35] that also obey to a non-linear equation from which it is possible to derive a kind of non-linear Schrödinger equation.

5.8.1 The two-slit experiment

Richard Feynman once said that all the "mystery" of quantum mechanics is conveyed by the one-particle two-slit experiment (see Figure 5.6). Indeed, it

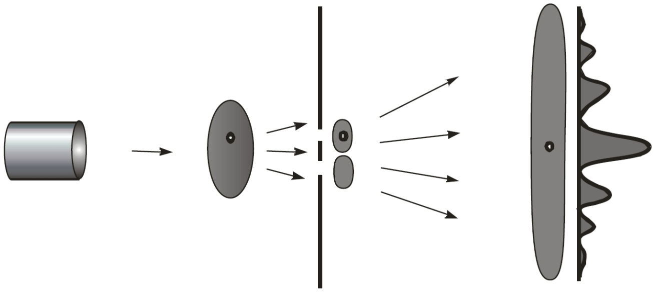

Figure 5.6. The two-slit experiment [32].

apparently shows, according to the orthodox interpretation, distinct behaviours *depending* on the observation apparatus. Thus, if two detectors are placed just after the slits, the conclusion is that the particle passes through one slit *or* the other. If a screen is placed sufficiently far away from the slits, then the conclusion is that the particle passes through one slit *and* the other accounting for the interference. The collapse of the wave function is invoked and reality appears to be created by the observation process, since two distinct processes are required to show up particle-like *or* wave-like properties. Never both simultaneously once the measuring processes are incompatible. Ultimately, the mystery is: how can an *indivisible* particle go simultaneously through both slits?

For the local-causal interpretation there is neither mystery nor collapses of the wave functions, and the reality, composed of waves *and* particles, is independent of the observer. The indivisible acron passes through one slit *or* the other, and the "pilot wave" through *both* slits. If the acron is detected right after one of the slits, then the theta wave from the other slit will follow its own way and should be possible to detect it. If not, both waves will interfere and the particle will be detected at the screen, where the intensity of the resulting wave is higher. In any case the wave-particle properties are always present, that is, they are not created by the observation process.

The last interpretation is identical to de Broglie's view, expounded during the 5th Solvay Conference in 1927, on the "pilot wave" idea that was taken as the basis of his *double-solution theory* and de Broglie-Bohm's hidden-variable formulation. Croca's approach is also, in particular aspects, inspired by de Broglie's work. Curiously, it is reported that, by 1928, de Broglie became converted to the orthodox view, presumably influenced by fellows (particularly Pauli) of the Copenhagen school. Later, however, he was interested in Bohm's work even writing, in 1957, the forward of Bohm's book *Causality and Chance in Modern Physics* [1, 4, 6].

5.8.2 Beyond Heisenberg's uncertainty relations

Heisenberg derived the uncertainty relations in 1927, based on Born's interpretation and transformation theory. His early interpretation considered the electrons essentially as corpuscle-like entities, and the relations as an instrumental impossibility of simultaneously specifying non-commuting observables. The uncertainty principle was, to him, the very basis of quantum theory. Bohr strongly disagreed on Heisenberg's viewpoint. Arguing that the indeterminacy relations can be exclusively derived from a pure Fourier analysis of wave packets (and that Heisenberg's analysis of the γ-ray microscope experiment was flawed) he defended that the true heart of the theory was the wave-particle duality, accordingly to his complementary principle. All the rest would come out from it [1, 4, 13]. As already referred to, Heisenberg soon adhered to Bohr's interpretation.

In the context of local wavelet analysis, however, more general uncertainty relations have been derived [32]. For position-momentum:

$$\Delta x^2 = \frac{h^2}{\Delta p_x^2 + h^2/\sigma_0^2}$$

and for time-energy:

$$\Delta t^2 = \frac{h^2}{\Delta E^2 + h^2/\sigma_0^2},$$

where σ_0 is the average width of the basic (or "mother") wavelet. If $\sigma_0 \to \infty$, then the wavelet tends to a monochromatic harmonic wave (Fourier analysis) and Heisenberg's relations are obtained.

The generalized relations allow for a wider spanning of the measurement spaces than Heisenberg's relations (see Figure 5.7). Consider the measurement

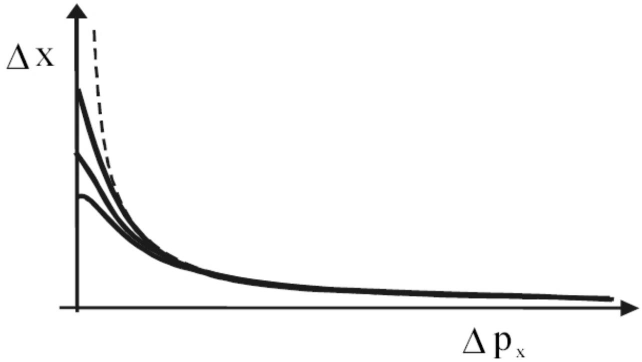

Figure 5.7. Spanning of position-momentum space: (solid lines) generalized relations for different values of the basic wavelet; (dashed line) Heisenberg's relation [32].

of the position and momentum of a particle by a common and by a tunnelling super-resolution microscope. The maximum momentum uncertainty for both is:

$$\Delta p_x = 2\frac{h}{\lambda}$$

For the common microscope, the maximum theoretical resolution is:

$$\Delta x = \frac{\lambda}{2},$$

therefore

$$\Delta x \Delta p_x = h,$$

that is Heisenberg's relation.

As for the super-resolution microscope, the resolution is at least,

$$\Delta x = \frac{\lambda}{50},$$

whence

$$\Delta x \Delta p_x = \frac{1}{25}h$$

which is in discrepancy with Heisenberg's relation but not with Croca's one.

5.9 Some issues

Like in any new theory, multifarious questions naturally turn out, particularly to us who never worked on the present approach. For example:

(i) If Ψ_1, Ψ_2, Ψ_3, ..., Ψ_n are solutions of Schrödinger's equation so is:

$$\Psi = \Psi_1 + \Psi_2 + \Psi_3 + \cdots + \Psi_n \quad \text{(superposition principle)}$$

The same is not true for the master non-linear equation. Then, how to compose the solutions? $\Psi = \Psi(\Psi_1, \Psi_2, \Psi_3, \ldots, \Psi_n)$ is not known in general.

(ii) How to incorporate spin and symmetry aspects?

(iii) The non-linear resolutions of the harmonic oscillator and the hydrogen atom are under progress, showing solutions other than the usual ones [36]. What is their meaning? Do they add new informations?

(iv) Experiments have been proposed to detect the θ waves. At least one of them has been performed though, apparently, not conclusive [32]. What is the expectable progress?

(v) Apart the new picture, the appealing interpretations and the more general uncertainty relations, will the heavy burden of solving non-linear equations, in complex chemical and physical problems, be rewarded for new and unexpected results not reachable by the orthodox linear theory?

(vi) The approach suggests the possibility of understanding gravitational phenomena [32]. Is it a route to unify quantum and general relativity theories?

5.10 Linearity versus non-linearity

The linearity of the orthodox theory has been questioned by various authors. Here, we only cite Steven Weinberg: "Quantum mechanics has had phenomenal successes in explaining the properties of particles and atoms and molecules, so we know that it is a very good approximation to the truth. The question then is whether there is some other logically possible theory whose predictions are very close but not quite the same as those of quantum mechanics ... It is striking that it has so far not been possible to find a logically consistent theory that is close to quantum mechanics, other than quantum mechanics itself ...

In inventing an alternative to quantum mechanics I fastened on the one general feature of quantum mechanics that has always seemed somewhat more arbitrary than others, its linearity …

This theoretical failure to find a plausible alternative to quantum mechanics, even more than the precise experimental verification of linearity, suggests to me that quantum mechanics is the way it is because *any small change in quantum mechanics would lead to logical absurdities*. If this is true, quantum mechanics may be a permanent part of physics. Indeed, quantum mechanics may survive not merely as an approximation to a deeper truth, in the way that Newton's theory of gravitation survives as an approximation to Einstein's general theory of relativity, but as a precisely valid feature of the final theory" [37].

The reason of the reported failure seems to be, in fact, that *any small change in quantum mechanics would lead to logical absurdities*. Indeed, the orthodox theory is based on a rigorous mathematical structure through a consistent set of postulates intrinsically based on linearity. In this strict context, it appears a complete statistical theory, though physically and philosophically anti-realist in many aspects. Therefore, amendments can lead to absurdities since the self-consistency of the theory structure might be broken. Thus, it seems that only an approach that changes the fundamental structure of the theory may fully avoid logical and physical contradictions.

5.11 Language, thought and perception

The spoken and pictorial languages are intimately related to the thought and perception processes. Quoting Lee Whorf: "We are thus introduced to a new principle of relativity, which holds that all observers are not led by the same physical evidence to the same picture of the universe, unless their linguistic backgrounds are similar, or can in some way be calibrated" [38].

For example, the language of the American Hopi Indians contains no reference to *time* either explicitly or implicitly. Yet, it is capable of accounting for all observable phenomena of the universe. Time is not one of the measurement observables that the Hopi Indians employ. They use other means to speak of the universe. Their language expresses their perception, and it does not include time.

On the other hand, time is not an observable in quantum mechanics, since there is no *time operator* in its structure. Curiously, for Kurt Gödel, "time does not really exist in any objective sense. It's not really out there in the world at all; it's our special mode, our own particular way of perceiving the world" [39].

Mathematics is also a language, simultaneously analytic and geometric (pictorial), more universal and suitable for a calibration (in Lee Whorf's sense). But will it be essentially primitive and intrinsically connected to our processes of thought and perception? Or a language that only attempts to express facts perceived *a priori*, even in the most abstract developments? Incidentally, what would be the representations of Schrödinger and non-linear master equations in the Hopi language?

These questions appear of the utmost importance, especially in the context of quantum mechanics since its language and interpretation problems fall right into the processes of perceiving and conveying the nature of physics.

5.12 Concluding remarks

In this digression we have addressed some approaches and interpretations of quantum theory. After all, there were more questions than answers but quoting J. Joubert: "it is better to debate a question without settling it than to settle a question without debating it" [1].

Some issues on the theory interpretation and unifications will certainly continue at stake, challenging researchers and philosophers, and stimulating new steps forward.

However, one point seems inescapable: *contraria sunt complementa*, as Bohr inscribed at the top of the *yin-yang* symbol in his Cote-of-Arms, when he was knighted (Order of the Elephant) in 1947.

The eventual spiritual anguish due to the implicit contradictions might, hopefully, be relieved by:

Do I contradict myself?
Well then, I contradict myself.

I am large, I contain multitudes.

(Walt Whitman)

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