Journées sur les Arithmétiques Faibles 35

Departamento de Matemática, Faculdade de Ciências da Universidade de Lisboa
6-7 June 2016

ABSTRACT BOOKLET

FCT Fundação para a Ciência e a Tecnologia
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CMAF CIO
CENTRO DE MATEMÁTICA, APLICAÇÕES FUNDAMENTAIS E INVESTIGAÇÃO OPERACIONAL

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Timetable

	JAF35 PROGRAMME OVERVIEW	
	6 June (Monday)	7 June (Tuesday)
9h00-9h30	Registration	
9h30-10h00		Ali Enayat
10h00-10h30	Niel Thapen	Arthur Milchior
10h30-11h00	Coffee-break	Coffee-break
11h00-11h30	Henri-Alex Esbelin	Daniel Graça
11h30-12h00	Manuel Loureiro	
12h00-12h30	Rasmus Blanck	Emil Jerabek
12h30-13h00		
13h00-13h30		
13h30-14h00		
14h00-14h30	Lunch	Lunch
14h30-15h00		
15h00-15h30	Isabel Oitavem	Tin Lok Wong
15h30-16h00	Michal Garlik	Michal Godziszewski
16h00-16h30	Coffee-break	Coffee-break
16h30-17h00		Joost Joosten
17h00-17h30	David Fernandez-Duque	Saeideh Bahrami
20h00-	Conference dinner	

INVITED TALKS:

- Ali Enayat: Playing with Fire: Interpreting PA with a full satisfaction predicate within PA
- David Fernández-Duque: From weak to strong arithmetic via reflection
- Emil Jeřábek: Diophantine formulas
- Isabel Oitavem: Applicative theories for the polynomial hierarchy of time and its levels
- Neil Thapen: A feasible set theory
- Tin Lok Wong: Upgrading the arithmetized completeness theorem

CONTRIBUTED TALKS:

- Rasmus Blanck: A characterisation of Π_1-conservativity over $I\Sigma_1$
- Henri-Alex Esbelin: Word problem vs modular counting in Δ_0
- Michal Garlík: Bounded arithmetic and a restricted reduced product construction
- Michał Godziszewski: Short elementary cuts in recursively saturated models of arithmetic
- Daniel Graça: Computability of the Lorenz attractor
- Saeideh HajiBahrami: The germomorphism group of a model of arithmetic
- Joost Joosten: Characterizations of interpretability in bounded arithmetic
- Manuel Loureiro: On the reverse mathematics of Lipschitz and Wadge determinacy
- Arthur Milchior: Undecidability of satisfiability of expansions of FO[$<$] over words with a FO[$+$]-definable set

Playing with fire: Interpreting PA with a full satisfaction predicate within PA

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Abstract

$\mathrm{PA}^{\mathrm{FS}}$ (Peano arithmetic with a full satisfaction class) is the theory extending PA with a finite set of axioms that together specify that S obeys the usual Tarski conditions for a satisfaction predicate for the ambient arithmetical structure (where S is a new binary predicate symbol added to the arithmetical language). The conservativity of $\mathrm{PA}^{\mathrm{FS}}$ over PA was established by Krajewski, Kotlarski, and Lachlan [KKL].

In this talk I will present a proof of the *interpretability* of $\mathrm{PA}^{\mathrm{FS}}$ in PA, using a novel arithmetization of the model-theoretic proof [EV] of the conservativity of $\mathrm{PA}^{\mathrm{FS}}$ over PA.

References

[EV] A. Enayat & A. Visser, *New constructions of satisfaction classes*, **Unifying the Philosophy of Truth** (edited by T. Achourioti, H. Galinon, J. Martínez Fernández, & K. Fujimoto), Springer, pp. 321-335, 2015.

[KKL] H. Kotlarski, S. Krajewski, & A.H. Lachlan, *Construction of satisfaction classes for nonstandard models*, **Canadian Mathematical Bulletin**. vol. 24 (1981), pp. 283–293.

June 7, 9h00-10h00

From weak to strong arithmetic via reflection

David Fernández-Duque
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Abstract

If T is a formal theory, local reflection over T is the scheme that states that if A is a theorem of T, then A must be true. More generally, uniform reflection allows parameters in A, and states that if T proves $A(n')$, where n' is the numeral of n, then $A(n)$ must hold. A classic result of Kreisel and Lévy states that Peano arithmetic is equivalent to primitive recursive arithmetic (PRA) together with the uniform reflection scheme for PRA.

In this talk, we will discuss extensions of this result to some of the 'big five' theories of reverse mathematics. To be precise, we will define reflection principles for variants of omega-logic with oracles. Then, we show that both the theory ATR_0 of arithmetical transfinite recursion, and $\Pi^1_1 - CA_0$ of comprehension for Π^1_1 formulas, are equivalent to such reflection principles over a weak base theory.

June 6, 16h30-17h30

Diophantine formulas

Emil Jeřábek
Institute of Mathematics of the Czech Academy of Sciences, Prague

Abstract

Standard undecidability theorems show that for any reasonable arithmetic theory T, the set of Π_1 consequences of T is undecidable; equivalently, the set of Σ_1 sentences satisfiable in a model of T is undecidable. Better yet, sufficiently strong theories T (in particular, theories extending $I\Delta_0 + EXP$, by a result of Gaifman and Dimitracopoulos) prove the MRDP theorem; for such theories, even the set D_T of *Diophantine equations* solvable in a model of T is undecidable.

What is the complexity of D_T for weaker theories T? On the one hand, Kaye proved that D_T is undecidable already for extensions of the theory IE_1^-, despite that it (presumably) does not formalize the MRDP theorem as such. On the other hand, it is a longstanding open problem whether D_T is decidable for $T = IOpen$ or $T = PA^-$. In this talk, we will consider an even weaker theory: Robinson's arithmetic Q. We will see that in contrast to stronger fragments of arithmetic, D_Q is decidable, and in fact, NP-complete. As a byproduct of the argument, we will obtain an explicit axiomatization of the universal fragment of Q.

June 7, 11h30-12h30

Applicative theories for the polynomial hierarchy of time and its levels

Isabel Oitavem[1]
CMA and DM, FCT-UNL

Abstract

We characterize the polynomial hierarchy of time (FPH) and its levels in a recursion-theoretic manner, in the vein of Cobham's characterization of FPtime. This is based on [1] and it combines monotonicity constraints with a known characterization of FPspace [2].

Given a function algebra, the main challenge concerning the design of a corresponding theory is, of course, to introduce an appropriate induction scheme which allows to prove properties for the functions under consideration.

In this talk we adopt applicative theories as underlying framework. Applicative theories are the first-order part of Feferman's system of explicit mathematics. They provide a very handy framework to formalize theories of different strength, including to characterize classes of computational complexity.

We describe applicative theories for FPH and its levels. In order to achieve this we incorporate the monotonicity constraints of the function algebras in the corresponding induction schemes [3].

References

[1] A. Ben-Amram, B. Loff and I. Oitavem, Monotonicity constraints in characterisations of PSPACE, Journal of Logic and Computation, 22, 2012, pp.179-195.

[2] I. Oitavem, New recursive characterizations of the elementary functions and the functions computable in polynomial space, Revista Matemática de la Universidad Complutense de Madrid, vol.10, n.1, 1997, pp.109-125.

[3] R Kahle and I. Oitavem, Applicative theories for the polynomial hierarchy of time and its levels, Annals of Pure and Applied Logic, 164/6, 2013, pp.663-675.

June 6, 14h30-15h30

[1] Research partially supported by the Portuguese Science Foundation, FCT, through the project *Hilbert's 24th problem*, PTDC/MHC-FIL/2583/2014, and UID/MAT/00297/2013 (*Centro de Matemática e Aplicações*).

A feasible set theory

Neil Thapen
Institute of Mathematics, Czech Academy of Sciences

Abstract

The Cobham recursive set functions (CRSF) are the closure of some basic functions under composition and a kind of limited recursion, analogous to Cobham's definition of polynomial time on binary strings. They generalize the notion of polynomial time to functions on arbitrary sets. In particular, they define exactly P when restricted to finite binary strings, and they correspond to a functions computed by uniform "polynomial size" infinite circuits.

I will describe a weak fragment of Kripke-Platek set theory analogous to Buss's S^1_2 and show, with some caveats, that CRSF contains exactly the provably recursive functions of this theory. This is joint work with Arnold Beckmann, Sam Buss, Sy Friedman and Moritz Mueller.

June 6, 9h30-10h30

Upgrading the Arithmetized Completeness Theorem

Tin Lok Wong
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Abstract

The Arithmetized Completeness Theorem (ACT) is a formalization of Gödel's Completeness Theorem for first-order logic in arithmetic. It provides a powerful method for constructing nonstandard models of first-order arithmetic. Recently, Ali Enayat (University of Gothenburg, Sweden) and I observed that the ACT is also useful in producing models of second-order arithmetic. I will demonstrate this by explaining how conservation theorems in the Simpson–Tanaka–Yamazaki paper (*Ann. Pure Appl. Logic* 118:87–114, 2002) can be proved using a combination of forcing and ACT techniques.

June 7, 14h30-15h30

A characterisation of Π_1-conservativity over $I\Sigma_1$

Rasmus Blanck
University of Gothenburg

Abstract

By putting together a number of classic results due to Orey, Hájek, Guaspari and Lindström we get the well known characterisation of Π_1- conservativity over extensions T of Peano arithmetic PA. In short, the following are equivalent for a sentence ϕ:

1. T+ϕ is Π_1-conservative over T,
2. T+ϕ is interpretable in T,
3. for each $n \in \omega$, T$\vdash$ $\mathrm{Con}_{(\mathrm{T}+\phi)\restriction n}$
4. every model of T can be end-extended to a model of T+ϕ,
5. every countable model of T can be end-extended to a model of T+ϕ,
6. for every countable model $\mathscr{M}$ of T, T+$\mathrm{Th}_{\Sigma_1}(\mathscr{M}) + \phi$ is consistent.

If we instead consider extensions T of $I\Sigma_1$, the characterisation breaks down. In this case, neither of 1 or 2 implies the other; we can never have 3 if T is finitely axiomatised; and regarding 4, it is not even known if every model of $I\Sigma_1$ has a proper end-extension to a model of $I\Sigma_1$.

In this talk, which reports on joint work with Ali Enayat, we show that it is possible to salvage parts of this characterisation for extensions of $I\Sigma_1$. The equivalence of 1, 5 and 6 can still be shown to hold, and we also present another equivalent condition, which is similar to 3, but phrased in terms of bounded provability.

June 6, 12h00-12h30

Word problem vs modular counting in Δ_0

Henri-Alex Esbelin ([2])

Abstract

Iterated products of elements of groups appear in several areas of complexity theory. Here we consider:

• the *Word Problem*: the data are words of the language (seen as a semi-group for concatenation) over the alphabet given by a finite (if it exists) set of generators of the group and their inverses; the considered complexity classes are generally defined in the Turing machines framework. An exemple of theorem in this area is:

Theorem. (Lipton and Zalcstein, [1]) The word problem for finitely generated free groups is solvable in $LOGSPACE$.

• the *counting modulo a (semi)-group*: the data is a function of domain $\mathbb{N}$ (eventually with parameters in the same set) and of codomain the given group; the complexity classes are generally classes of the Grzegorczyk hierarchy or are defined in the weak arithmetic framework. An exemple of theorem in this area is:

Theorem. (Esbelin [2]) Let us consider $\left\{\begin{array}{ccc} \mathbb{N} & \to & SL(2,\mathbb{N}) \\ i & \to & \begin{pmatrix} a(i) & b(i) \\ c(i) & d(i) \end{pmatrix} \end{array}\right.$ where the relation of variables u, v, w, z and i defined by $\begin{pmatrix} u & v \\ w & z \end{pmatrix} = \begin{pmatrix} a(i) & b(i) \\ c(i) & d(i) \end{pmatrix}$ is Δ_0–definable. Then the relation of

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variables u, v, w, z and y defined by $\begin{pmatrix} u & v \\ w & z \end{pmatrix} = \begin{pmatrix} a(0) & b(0) \\ c(0) & d(0) \end{pmatrix} \begin{pmatrix} a(1) & b(1) \\ c(1) & d(1) \end{pmatrix} \dots \begin{pmatrix} a(y) & b(y) \\ c(y) & d(y) \end{pmatrix}$ is $\Delta_0^{\sharp\mathbb{N}}-$definable.

We intend to compare some of the results in both areas.

Let G be a group with a finite set of generators $\Gamma = \{\gamma_1, ..., \gamma_d\}$. Let us denote $X = \{\gamma_1, ..., \gamma_d\} \cup \{\gamma_1^{-1}, ..., \gamma_d^{-1}\}$.
Definition. We say that a function $f : \mathbb{N}^k \mapsto X$ is a Δ_0-function iff for all $\gamma \in X$, the relation of variables $\vec{x}$ defined by $f(\vec{x}) = \gamma$ is Δ_0-definable.
Definition. We say that Δ_0 is closed under weak counting modulo G iff for all Δ_0-function f of variables $\vec{x}$ and i and for all $\gamma \in X$, the relation $f(\vec{x}, 0).f(\vec{x}, 1)....f(\vec{x}, z) = \gamma$ is Δ_0-definable.
Definition. We say that Δ_0 is closed under weak counting of logarithmic length modulo G iff for all Δ_0-function f and for all $\gamma \in X$, the relation $f(\vec{x}, 0).f(\vec{x}, 1)....f(\vec{x}, \lfloor \log_2(z) \rfloor) = \gamma$ is Δ_0-definable.

The same definitions are convenient for all complexity classes of subsets of power of $\mathbb{N}$ such as $LOGSPACE$ (through a binary coding of the integers) or $\Delta_0^{\sharp\mathbb{N}}$ instead of Δ_0. For informations about these classes, see [3].

A theorem comparing complexity scales of the areas *word problem* and *counting modulo a group* is the following:

Theorem. Let G be a finitely generated group.
$LOGSPACE$ is closed under weak counting of logarithmic length modulo G
is equivalent to
the word problem for G is solvable in $LOGSPACE$
implies
Δ_0 is closed under weak counting of logarithmic length modulo G.

References

[1] R. J. Lipton and Y. Zalcstein, *Word Problems Solvable in Logspace*, Journal of the ACM (JACM) JACM Homepage archive Volume 24 Issue 3, July 1977 pp. 522-526

[2] H.-A. Esbelin, *Counting in Δ_0 modulo Matrix Monoids*, communication, JAF 33

[3] H.-A. Esbelin and M. More, *Rudimentary Relations and Primitive Recursion: A Toolbox*, Theoretical Computer Science 193(1-2):129-148

June 6, 11h00-11h30

Bounded arithmetic and a restricted reduced product construction

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Abstract

We present a construction of models of bounded arithmetic that yields nonelementary extensions but does not introduce new lengths. The construction has the form of a restricted reduced product. As an application we show that under the assumption of the existence of a one-way permutation g hard against polynomial-size circuits, $strictR_2^1(g)$ is weaker than $R_2^1(g)$. In particular, if such a permutation is definable by a term in the language of R_2^1, then $strictR_2^1$ is weaker than R_2^1.

June 6, 15h30-16h00

Short elementary cuts in recursively saturated models of arithmetic

Michał Tomasz Godziszewski
University of Warsaw

Abstract

We study certain model-theoretic properties of countable recursively saturated models of arithmetic. Our primary inspiration for examining mathematical features of such structures, and recursively saturated in particular, is that every countable recursively saturated model of Peano Arithmetic supports a great variety of nonstandard satisfaction classes that can serve as models for formal theories of truth - those models allow to investigate the role of arithmetic induction in semantic considerations. In the other direction, nonstandard satisfaction classes are used as a tool in model theoretic constructions providing answers to questions in the model theory of formal arithmetic and often allow to solve problems that do not explicitly involve nonstandard semantics.

Due to the fact that a for a countable model of arithmetic, it is equivalent to admit a full satisfaction class (i.e. satisfy the formal theory of compositional truth) and to be recursively saturated, the project can be thought of as an investigation into structure of possible interpretations for theory of compositional, arithmetical truth. It needs to be underlined that the purpose of our research is to examine model-theoretic but purely arithmetical properties of models admitting satisfaction classes. In particular, we study various substructures of recursively saturated models of PA, and we focus on the cofinal extensions of models of PA.

First-order theories of pairs (N, M), where $N \models PA$ and M is an elementary cofinal submodel of M reveal great diversity and demand systematic study. The case of models admitting satisfaction classes is of particular interest in this respect: all countable recursively saturated models of PA have continuum many nonisomorphic cofinal submodels, and after acknowledging the variety of the abovementioned pairs for N being countable recursively saturated, the next goal is to consider isomorphism types and first-order theories for pairs of models (N, M) for a fixed countable recursively saturated model N and a fixed isomorphism type of M. The method that has already been shown quite effective in this direction is the method of *gaps* (also called *skies*). We present briefly the gap terminology and explain why it is useful.

Skolem terms, also called simply definable functions[3], are paramter-free definable and PA-provably total functions. Let $\mathscr{M}$ be a nonstandard model of arithmetic and let $\mathscr{F}$ be some family of Skolem terms $f : M \to M$ such that $\forall x, y \in M\ x < y \Rightarrow x \leq f(x) \leq f(y)$. There is a partition of M into sets, which we call $\mathscr{F}$-gaps. For any $a \in M$, $gap_{\mathscr{F}}(a)$ is the smallest set $C \subseteq M$ such that $a \in C$ and: $\forall b \in C \forall f \in \mathscr{F} \forall x \in M\ b \leq x \leq f(b) \vee x \leq b \leq f(x)) \Rightarrow x \in C$. This is a natural generalization of an idea of partitioning the universe of a nonstandard model into $\mathbb{Z}$-blocks around each element (then, each such block is $gap_{\mathscr{F}}(a)$ for some a, where $\mathscr{F}$ consists only of the successor function s).

The gap of $a \in M$, denoted by $gap(a)$, is the $\mathscr{F}$-gap of a, where $\mathscr{F}$ is the family of **all** such definable functions, i.e. $\mathscr{F} = \{f : M \to M : f$ is definable and $\forall x, y \in M\ x < y \Rightarrow x \leq f(x) \leq f(y)\}$.

Every model $\mathscr{M}$ has the least gap, the $gap(0)$. Let $A \subseteq M$. Then, we denote $sup(A) = \{x \in M : \exists y \in A\ x \leq y\}$. If for some $a \in M$, $M = sup(gap(a))$, then we call $gap(a)$ the **last gap of** $\mathscr{M}$. A model with a last gap is called **short**. If $\mathscr{M} \preceq_{cut} \mathscr{N}$ (i.e. $\mathscr{M}$ is an elementary cut of $\mathscr{N}$), we say that $\mathscr{M}$ is **short elementary cut** of $\mathscr{N}$ if $\mathscr{M}$ is short - in other words, if by $Scl(a)$ we denote the set $\{t(a) :\ t$ is a Skolem term of PA$\}$ $\mathscr{M}$ is short if there is such an element $a \in M$ that its Skolem closure in $\mathscr{M}$ is cofinal in $\mathscr{M}$, i.e. for all $x \in M$ there is $b \in Scl(a)$ such that $x <_{\mathscr{M}} b$. An elementary cut is **coshort** if $\mathscr{N} \setminus \mathscr{M}$ has the least gap, i.e. there is $a \in N \setminus M$ s.t. $M = inf(gap(a))$, where $inf(A) = \{x \in M : \forall y \in A\ x \leq y\}$.

Now, to clarify the gap terminology, if we put: $\mathscr{M}(a) = sup(Scl(a))$, and $\mathscr{M}[a] = \{b \in M : \forall t \in Scl(b)\ t(b) < a\}$, then the set $[a] = \mathscr{M}(a) \setminus \mathscr{M}[a]$ is exactly the $gap(a)$. It can be shown that $\mathscr{M}(a)$ is the smallest elementary cut of $\mathscr{M}$ containing a, and that $\mathscr{M}[a]$ is empty if and only if everey elementary cut of $\mathscr{M}$ contains a. Gap terminology is particularly useful in the study of recursively saturated models of PA (see e.g. [5] for a reference to many methods and properties).

One of the interesting and natural questions conerning *pairs* for countable recursively saturated models of arithmetic and its cofinal submodels is the following *big* question of our particular interest:

Let $\mathscr{M} \models PA$ be a countable recursively saturated model and let $\mathscr{K}, \mathscr{K}'$ be elementary cuts of $\mathscr{M}$. Suppose that $(\mathscr{M}, \mathscr{K}) \equiv (\mathscr{M}, \mathscr{K}')$. Does it follow that $(\mathscr{M}, \mathscr{K}) \cong (\mathscr{M}, \mathscr{K}')$?

[3] with a slight abuse of terminology that is unimportant to our investigations

Another way to put it is: under what conditions, does the identity of theories of such pairs imply their isomoprhism? It is known that the answer to the *big* question above is negative, if $\mathscr{K}$ and $\mathscr{K}'$ in question are coshort, as shown by R. Kossak and J. Schmerl in [6]. However, it remains open (and is considered to be difficult) what is the answer for the case in which $\mathscr{K}$ and $\mathscr{K}'$ are short elementary cuts of $\mathscr{M}$, i.e. are of the form $\mathscr{M}(a)$ and $\mathscr{M}(b)$ for some $a, b \in M$.

Since it is not hard to prove the equivalence that there exists an automorphism of such $\mathscr{M}$ if and only if $tp(a) = tp(b)$ (in the purely arithmetical language $\mathscr{L}$), where $tp(a) = \{\varphi(x) : \mathscr{M} \models \varphi(a)\}$ is the set of formulae satisfied in $\mathscr{M}$ by $a \in M$, the natural way to proceed is to consider the definable sets in $(\mathscr{M}, \mathscr{M}(a))$ and complete types realized in the last gap of $\mathscr{M}$. We might then first ask under what circumstances there is an element $c \in gap(a)$ such that $tp(c) \in Def(\mathscr{M}, \mathscr{M}(a))$ for $\mathscr{M}$ being a countable recursively saturated model of PA.

Using results of Smorynski from [12] and working with gaps and standard systems $SSy(\mathscr{M})$ of $\mathscr{M}$, i.e. the family of all subsets of $\mathbb{N}$ that are coded in $\mathscr{M}$[4] we show that

Theorem 1 (Tin Lok Wong, MTG). *Let $\mathscr{M} \models PA$ be a countable recursively saturated model and let $a, b \in M$. Suppose that $(\mathscr{M}, \mathscr{M}(a)) \equiv (\mathscr{M}, \mathscr{M}(b))$ (recall that $\mathscr{M}(a)$ and $\mathscr{M}(b)$ are short). If $SSy(\mathscr{M}) \subseteq Def(\mathbb{N})$, then $(\mathscr{M}, \mathscr{M}(a)) \cong (\mathscr{M}, \mathscr{M}(b))$.*

As the project is essentially *in progress*, we end with perspective paths for further work.

The conceptual import of the result is that taking a nonstandard model of compositional truth such that all its coded sets are already definable in the standard model, we are able to identify isomorphic cofinal short elementary cuts of the model just by looking on the arithmetical theory of both pairs considered.

References

[1] J. Barwise and J. Schlipf, *An introduction to recursively saturated and resplendent models*, **Journal of Symbolic Logic** 41 (1976), 531-536.

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[14] C. Smorynski, *Recursively saturated nonstandard models of arithmetic*, **Journal of Symbolic Logic** 46 (1981), 259 - 286.

June 7, 15h30-16h00

[4]It turns out that the standard system tells you a lot about the model; for example, any two countable recursively saturated models of the same completion of PA with the same standard system are isomorphic.

Computability of the Lorenz attractor

Daniel Graça
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Abstract

Computable analysis provides a framework where computational problems over the reals can be studied from a computability and computational complexity perspective. In this talk we analyze, within the context of computable analysis, a problem suggested by the Fields medalist Steve Smale in his list of open mathematical problems for the next century. This problem asks whether the Lorenz attractor exists. The Lorenz attractor is the structure to where the trajectories of the Lorenz model, first studied by E. N. Lorenz in 1963, converge and provides one of the first examples of a candidate of a strange attractor. The Lorenz attractor is rather difficult to study from a mathematical point of view and its own existence was not proved until recently. To make the analysis of the Lorenz model more amenable, a geometric model was introduced and it was recently shown in 2002 by Tucker, using a computer-assisted proof, that the Lorenz system behaves like the geometric model and hence has a strange attractor. In this talk we analyze the geometric Lorenz model from a computability perspective and show that its associated strange attractor is computable, thus showing that the Lorenz attractor is also computable. We will also show that the Lorenz attractor has an associated computable measure.

June 7, 11h00-11h30

The Germomorphism Group of a Model of Arithmetic

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Abstract

The germomorphism group of a nonstandard model $\mathscr{M}$ of PA (Peano Arithmetic) is defined as follows. Let $\mathscr{F}$ denote the collection of functions $f : I \to J$, where both I and J are cuts of $\mathscr{M}$ (i.e., initial segments of $\mathscr{M}$ that have no last element) such that f is an embedding (i.e., f is injective and operation preserving). Note that the collection of automorphisms of $\mathscr{M}$ is a subset of $\mathscr{F}$. The germomorphism group of $\mathscr{M}$, denoted $\mathbf{G}(\mathscr{M})$, is the result of modding out $\mathscr{F}$ by the equivalence relation $\sim$ defined by: $f \sim g$ iff there is a nonstandard $m \in M$ such that $f(x) = g(x)$ for all $x \leqslant m$. The group operation of $\mathbf{G}(\mathscr{M})$ is the obvious one induced by the composition operation. This group was introduced by V. Shavrukov who asked whether $|\mathbf{G}(\mathscr{M})| > 1$ for some model $\mathscr{M}$ of PA. In this talk we outline a proof of the following theorem, due to A. Enayat, that provides a strong positive answer to Shavrukov's question:

Theorem. $|\mathbf{G}(\mathscr{M})| = 2^{\aleph_0}$ *for every countable nonstandard model* $\mathscr{M}$ *of* PA.

This is a joint work with Ali Enayat.

June 7, 17h00-17h30

Characterizations of interpretability in bounded arithmetic

Joost J. Joosten
Dept. Lògica, Història i Filosofia de la Ciència - Universitat de Barcelona, Spain

Abstract

This paper deals with three tools to compare proof- theoretic strength of formal arithmetical theories: interpretability, Π^0_1- conservativity and proving restricted consistency. It is well known that under certain conditions these three notions are equivalent and this equivalence is often referred to as the Orey-Hájek characterization of interpretability.

In this paper we look with detail at the various implications in the Orey-Hájek characterization and study what conditions are needed for these implications and in what (weak) meta-theory the characterizations can be formalized.

June 7, 16h30-17h00

On the reverse mathematics of Lipschitz and Wadge determinacy

Manuel José S. Loureiro
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Abstract

The reverse mathematics of Gale-Stewart determinacy has been widely investigated over the last forty years, providing us with a fairly detailed picture of the strength of Gale-Stewart determinacy principles in terms of subsystems of second order arithmetic. In contrast, the situation for Lipschitz and Wadge determinacy is completely different. It is known that full second order arithmetic (Z_2) can prove that all Borel Lipschitz and Wadge games are determined (a result of A. Louveau and J. Saint Raymond). But there is no detailed analysis of the exact strength of Lipschitz or Wadge determinacy principles in terms of subsystems of Z_2.

The present work is a first step in order to fill this gap. We give direct formalizations of Lipschitz and Wadge games within Z_2, and we investigate the reverse mathematics of Lipschitz and Wadge determinacy, as well as the tightly related semilinear order principle, for the first levels of the Borel hierarchy. Most remarkably, we fully characterize ACA_0 in terms of Lipschitz determinacy for difference of closed sets in Cantor space; as well as ATR_0 in terms of Lipschitz determinacy for clopen (or open) sets in Baire space.

This is joint work with Andrés Cordón-Franco (Seville) and F. Félix Lara-Martín (Seville).

June 6, 11h30-12h00

Undecidability of Satisfiability of Expansions of FO[$<$] over Words with a FO[$+$]-definable set

Arthur Milchior[5]

Abstract

Two new characterizations of FO[$<$,mod]-definable sets, i.e. sets of integers definable in first-order logic with the order relation and modular relations, are provided. Those characterizations are used to prove that satisfiability of first-order logic over words with an order relation and a FO[$+$]-definable set that is not FO[$<$,mod]-definable is undecidable.

June 7, 10h00-10h30

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