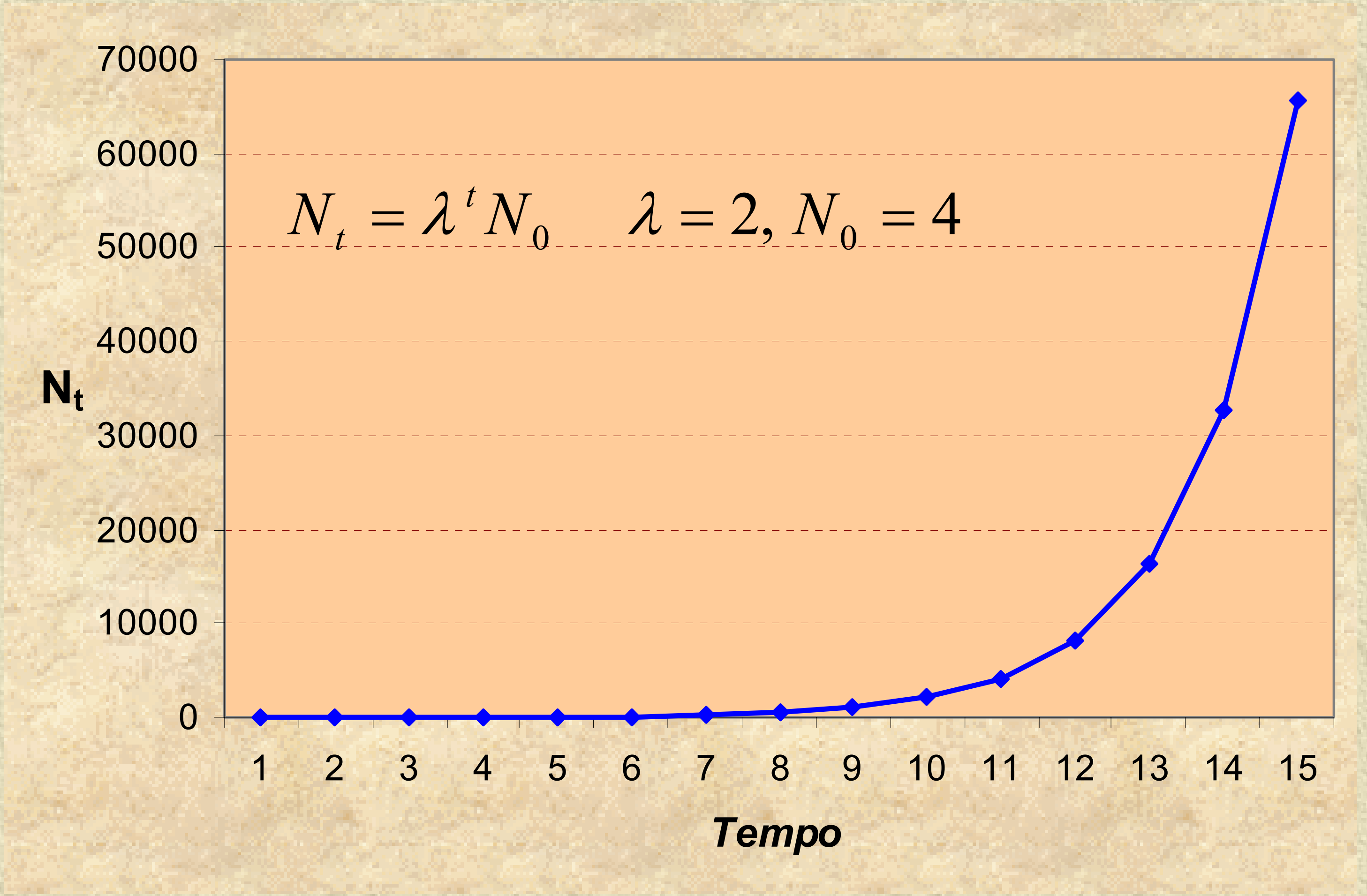
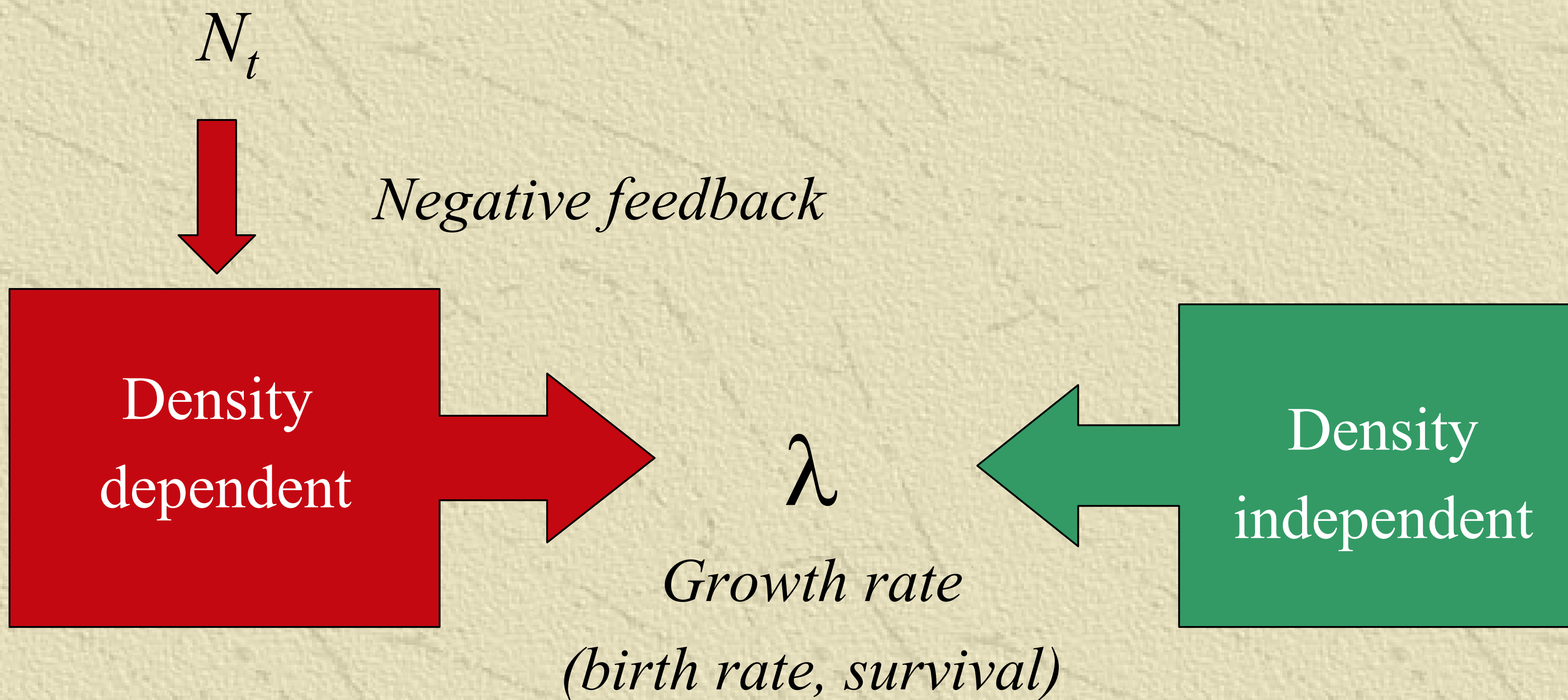
# *Exponential growth can't last forever*



# *Regulation factors*



# *Mechanisms that may induce density dependent regulation*

Limited food resources:

– Less consumption *per capita*, longer time periods searching for prey, with longer exposition to predators (affects S and b)

Less space:

-Smaller average territory or greater number of individuals without territory

Greater predator and/or parasite pressure:

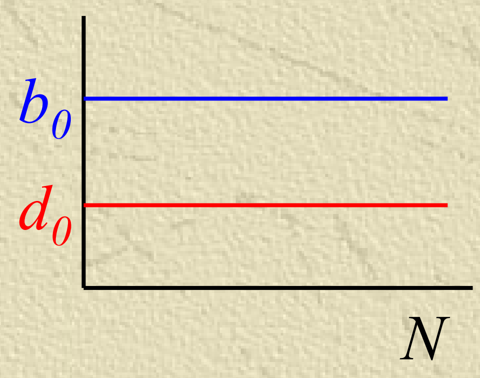
-Predators “shift” to denser prey populations; greater incidence of infectious diseases.

Greater use of marginal habitats of lesser quality

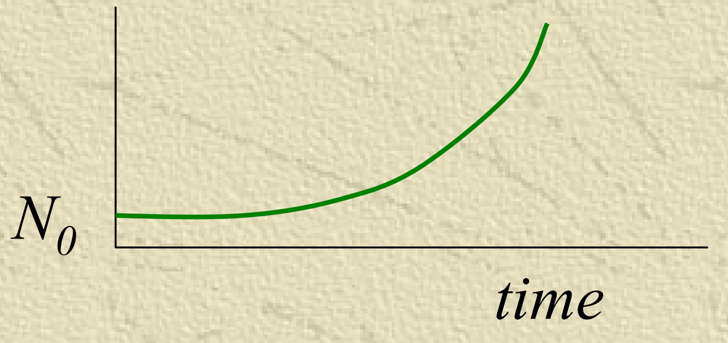
*etc. etc...*

# Exponential growth: constant *b* and *d*

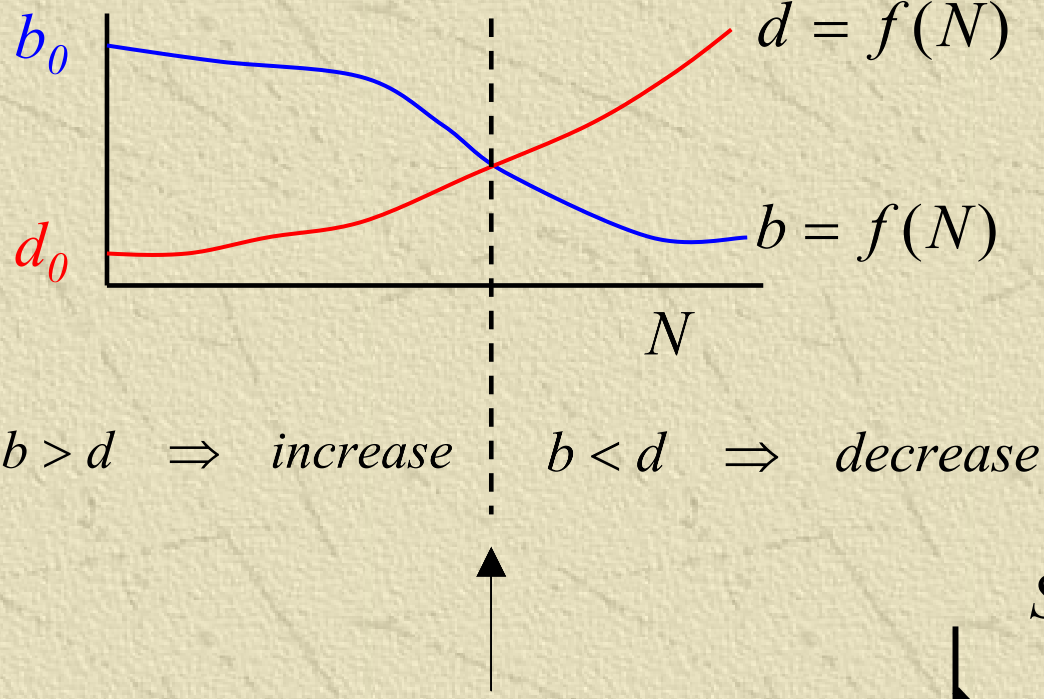
$N_t = N_0 e^{rt}$ $r = b_0 - d_0$ constants




$b_0 > d_0 \Rightarrow r > 0 \Rightarrow$ exponential growth

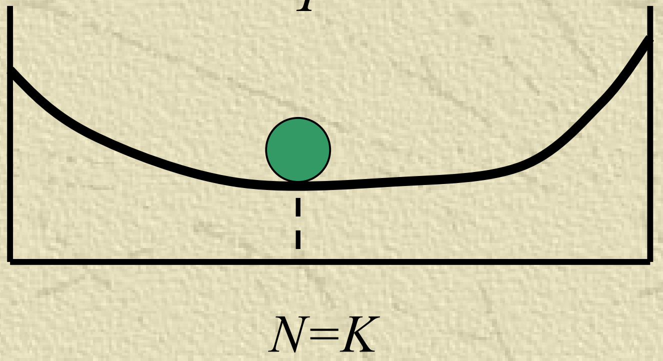


# *Density dependent regulation*



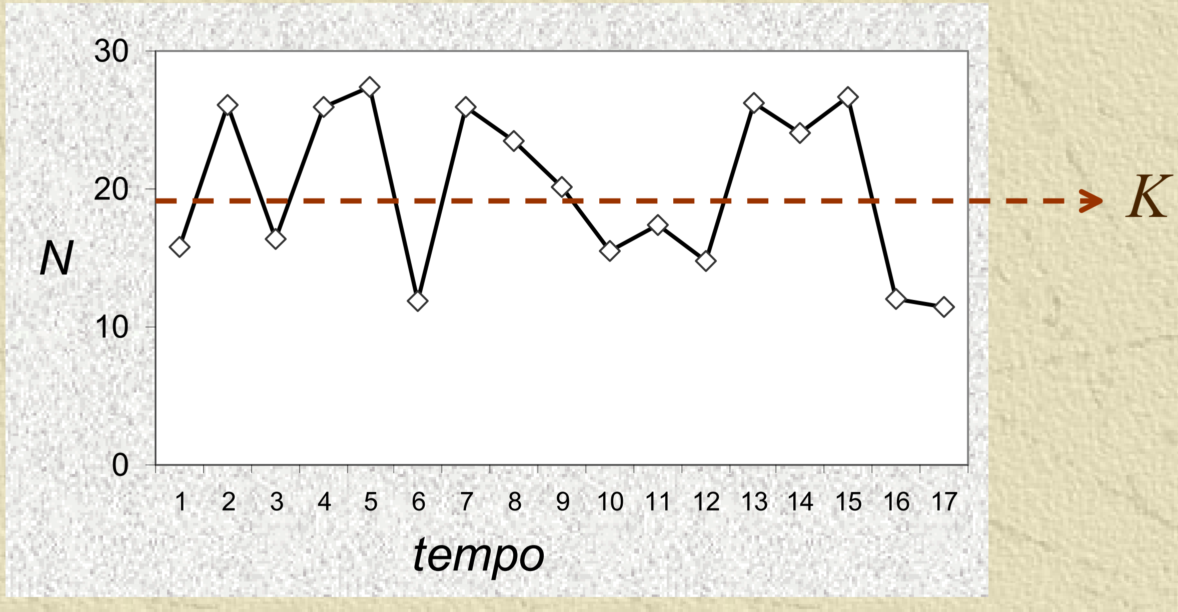

*Equilibrium, N = K*

*Stable equilibrium*



# *Carrying Capacity, K*

***Carrying capacity*** ≈ Population density which is sustained by the resources available



# *How to model density-dependence*

1. State the mechanisms of density dependence explicitly

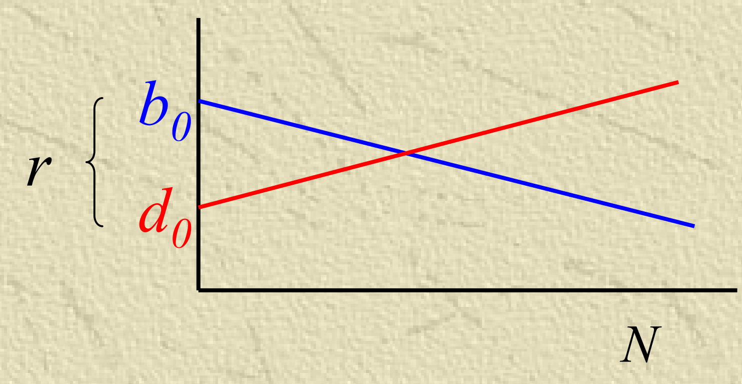
   Example: what are the mechanisms of

   intra-specific competition ?

2. Assume simple analytical functions for $b=f(N)$ and $d=f(N)$

   *Etc.*

# *Continuous breeding*




$$r = b_0 - d_0$$

$$d_t = d_0 + qN_t$$

$$b_t = b_0 - pN_t$$

Substituting in $$\frac{dN}{dt} = (b_t - d_t)N_t$$

we get $$\frac{dN}{dt} = [(b_0 - pN_t) - (d_0 + qN_t)]N_t$$

$$\frac{dN}{dt} = [(b_0 - d_0) - (p + q)N_t]N_t$$

# *Introducing K*

$N \longrightarrow K$

*At K, dN/dt = 0*

when does

$$\frac{dN}{dt} = 0 \quad ?$$

$$\frac{dN}{dt} = \left[r - (p+q)N_t\right]N_t$$

$N_t = 0$

*Trivial equilibrium*

$$N_t = \frac{r}{p+q}$$

Non-trivial equilibrium

K itself

# The logistic equation of continuos breeders *(Verhulst, 1838)*

$$K = \frac{r}{p+q} \quad \therefore \quad p+q = \frac{r}{K}$$

*Substituting here*

$$\frac{dN}{dt} = \left[r - (p+q)N_t\right]N_t$$

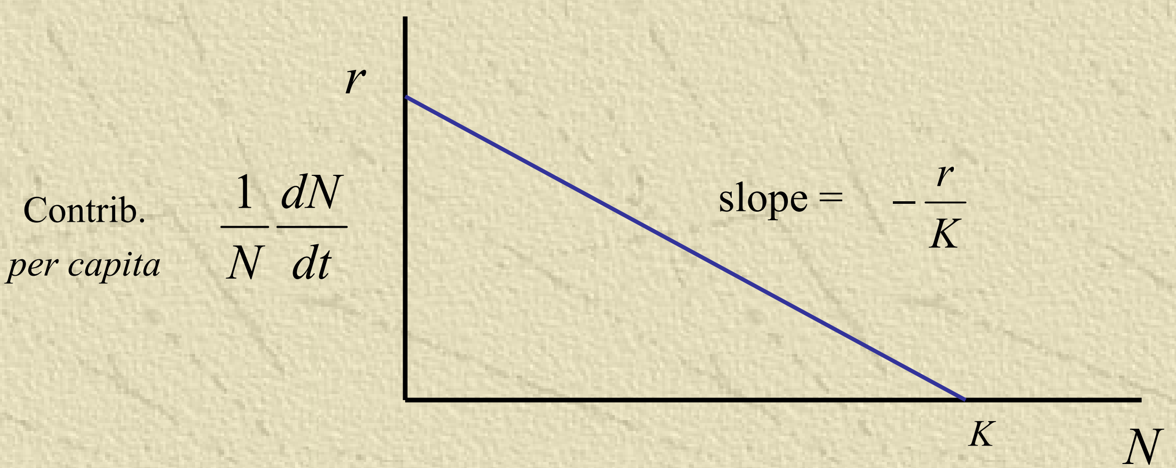
$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

***Unregulated growth*** ***Regulating factor***

# Growth *per capita*

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) \equiv \frac{1}{N}\frac{dN}{dt} = r - \frac{r}{K}N$$

Contribution of 1 individual for population growth

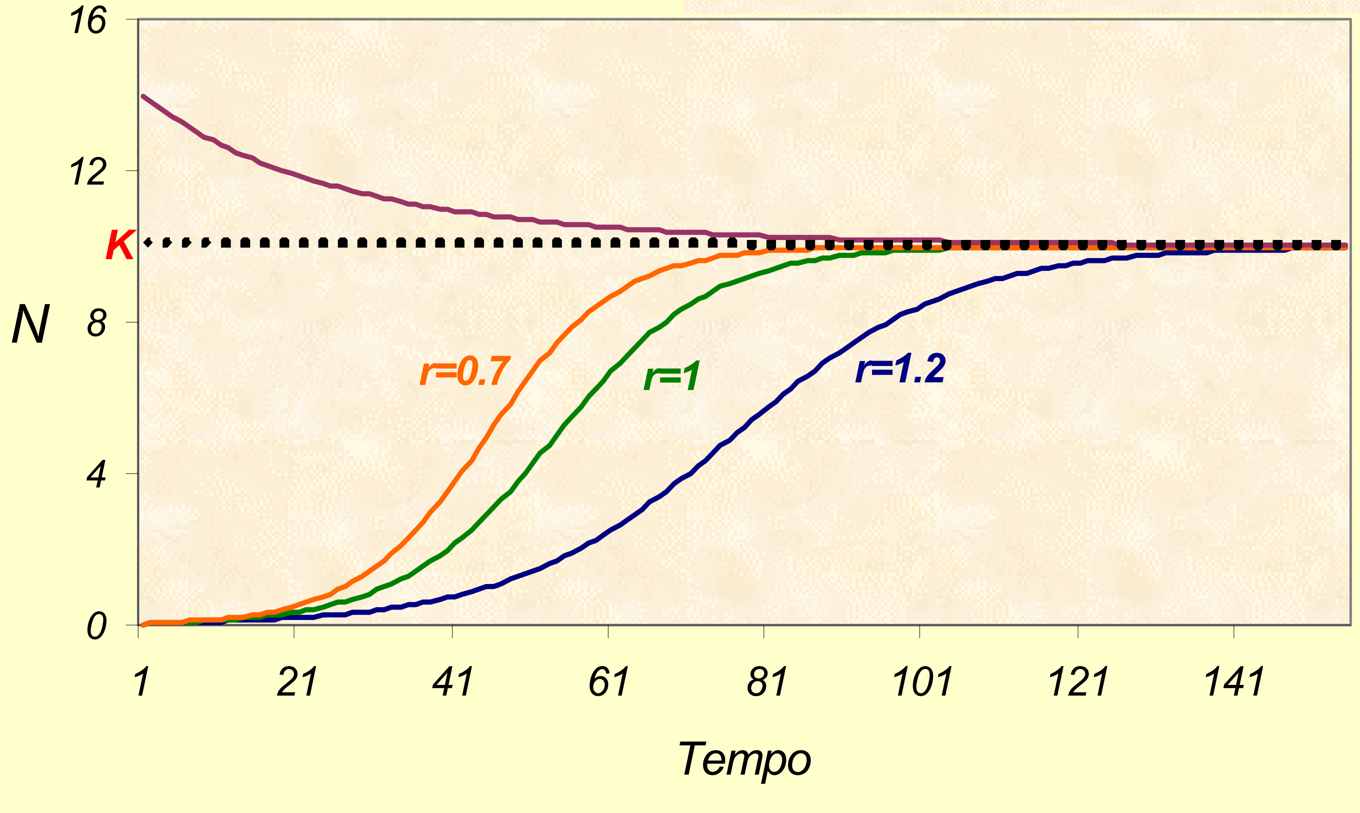


# *Solution of the logistic equation*

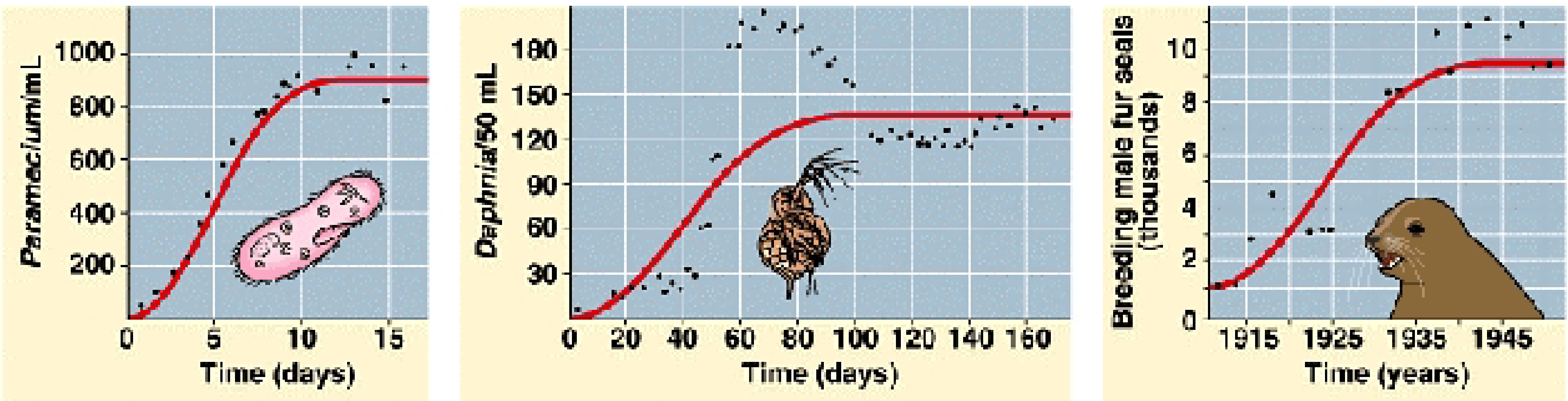
$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

*Solution:*

$$N_t = \frac{KN_0}{N_0 + (K - N_0)e^{-rt}}$$



# Figure 52.15 Examples of logistic population growth




(a) A *Paramecium* population in laboratory culture

(b) A *Daphnia* population in laboratory culture

(c) A fur seal (*Callorhinus ursinus*) population on St. Paul Island, Alaska