## The muskox revisited



## Variability in $\lambda$

Rate $\lambda=N_{t+1} / N_{t}$ changed every year around the mean (dashed line)


## Assumptions when projecting from $\mathbf{N}_{t}$ to $\mathbf{N}_{t+1}$

1. In $N_{t+1}=\lambda N_{t}$, we may use mean $\lambda$

Environmental variability may be neglected
2. The population is large enough for us to ignore individual variations in breeding and survival at the individual level.

We may neglect demographic stochasticity

## Demographic stochasticity (DS)

Birth an survival take place at the individual level and are random phenomena.

It is not possible to tell exactly how many individuals are going to be born or die between $t$ and $t+1$.

Randomness in survival and reproduction at the individual level is called:

Demographic stochasticity

## When are we to worry about DS ?

If the population is very small $\rightarrow$ greater risk of extinction for merely random processes

The population may go extinct even if $r>0$ !
Exples: Colonization
Populations at the edge of the species distribution Exploited populations (hunting and fisheries)

## Deterministic and stochastic models

## Deterministic

The parameters are constants (exple: $\lambda$ is constant)
Same initial conditions lead to the same results

$$
\left(N_{0}, n\right) \longrightarrow\left(N_{n}\right) \text { single results }
$$

Stochastic, random or probabilistic Parameters take values drawn from pre-defined probability distributions

Same initial conditions do NOT lead to the same results
$\left(N_{0}, n\right) \longrightarrow\left(N_{n}^{\prime}, N_{n}{ }^{\prime}, N_{n}{ }^{\prime \prime \prime}, \ldots\right)$ Distribution of results

## Probabilities of surviving and number of newborns from $t$ to $t+1$

$$
\lambda=f\left(\mathrm{~S}_{\mathrm{t}}, \mathrm{~b}_{\mathrm{t}}\right)
$$

Apply to the individual


$$
\mathrm{b}_{\mathrm{t}} \mathrm{St}
$$



## Parameters for the muskox


$S_{t}=0.921$ Probability of survival between $t$ and $t+1$
$b_{\mathrm{t}}=0.246$ Probability of 1 newborn at t
$b_{t} S_{t}=0.227$ Probability that the newborn is alive at $t+1$

Nunivak Island
31 individuals (1936)


## Simulating ES (survival)

$X$ is a prn drawn from a standard uniform distribution


Repeat for every animal

## Simulating ES (births)

## Repeat, simultaneously, for the 31 animals:

$Y$ is a prn drawn from a standard uniform distribution


## Replicates

## For 31 animals



Does this animal beget a surviving newborn ?
Does the animal survive?

## Repeat 10, 100, 1000, ... times

Each replicate yields a different number of individuals in $t+1$ !

## Simulations: $\mathrm{N}_{\mathrm{t}}=31,50$ replicates

Deterministic model:
$N_{t}=31$
$N_{t+1}=\lambda N_{t}=(1.148 \times 31)=35.6$

Stochastic model


## Questions addressed by the stochastic model

For example:

Probability of $N_{t+1} \leq 31$ individuals?
Probability that the population grows 10\% or more?

Probability of extinction?

Etc.

## Risk curve for period (t, t+1)



## "Risk"

Risk = Probability that an adverse event takes place in a given time period

Exples:
Population becomes smaller than $x$ individuals within the next 3 years
A pest outbreak takes place before 2010
Population becomes extinct in 100 years

## Simulating ES (births)

## Repeat, for every animal:

$X$ is number of newborns per individual in $(t, t+1)$
$b$ is the mean number of newborns per individual in $(t, t+1)$ $P(X=K) \cap \operatorname{Poisson}(b)$

$X$ is obtained by drawing Poisson(b) prn's $\quad X=0,1,2,3, \ldots$
(There are algorithms for this)
$\square$
$t$
$t+1$

