The shapes of water

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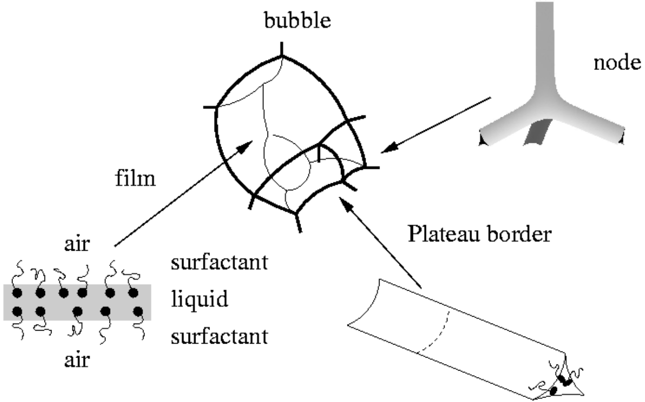
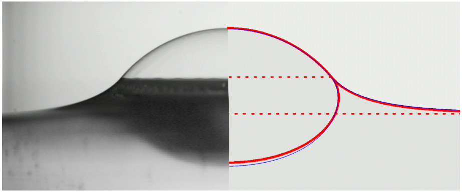
Consider a foam

- A foam consists of pockets, called cells or bubbles, of gas or liquid enclosed in liquid – liquid foams – or solid – solid foams.
- In liquid foams, liquid is distributed over films, Plateau borders (PBs), and nodes.



Bubble bubble, toil and trouble

- The building blocks of foams are bubbles.
- Previously we studied the shapes of bubbles on a **liquid** syrface:



Teixeira *et al.*. Langmuir **31**, 13708–13717 (2015).

- On a **solid** surfzce, they can have unusual shapes:

- . . . but curved geometries are difficult.

Consider a *confined* foam

- In confined foams there exist wall PBs, or menisci, where the films meet the confining substrates.

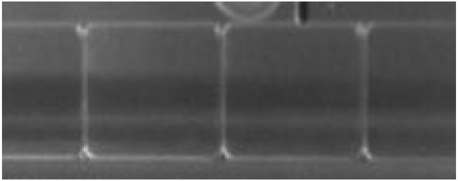


Photo courtesy of M. F. Vaz.

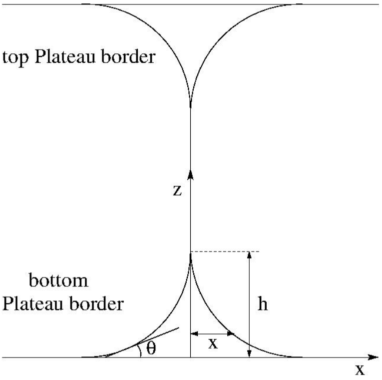
- One usually assumes tha the liquid wets the substrates completely, but this need not be so.
- What is the shape of a PB on a surface of a given wettability (i.e., contact angle θ_c)? **Can that surface support a foam?**
- This is important for firefighting foams, containers for foamy foodstuffs, etc.

Theory: model PB and the Young-Laplace equation

Young-Laplace equation for PBs where flat film meets planar substrate:

$$\left[1+\left(\frac{dx}{dz}\right)^2\right]^{-3/2}\frac{d^2x}{dz^2}=-\frac{\Delta p}{\gamma}$$

$\Delta p = [p_b(z=0)-\rho g z]-p_a$ is pressure difference across the interface ($\propto$ curvature), γ is liquid-gas surface tension.



Boundary conditions:

1. $dx/dz(z=0)=-\cot\theta_c$ (solid substrate);
2. $dz/dx(x=0)=+\infty$ (PB apex).

Theory: solving the Young-Laplace equation

- Rewrite equation in terms of film inclination $\theta(z)$: boundary conditions are $\theta(0) = \theta_c$, $\theta(h) = \pi/2$.
- Assume hydrostatic PBs, normalise lengths by h and introduce Bond number $\mathrm{Bo} = \rho g h^2/\gamma$.
- Analytically exact solutions for bottom $(+)$ and top $(-)$ PBs:

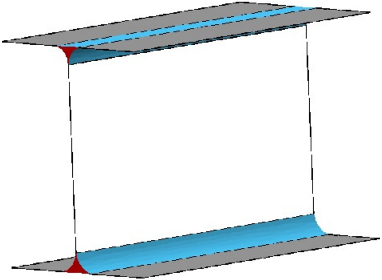
$$x'(z') = \int_{z'}^{1} \frac{(1 - z'')\left(\cos\theta_c \pm \frac{\mathrm{Bo}}{2} z''\right) dz''}{\left[1 - (1 - z'')^2 \left(\cos\theta_c \pm \frac{\mathrm{Bo}}{2} z''\right)^2\right]^{1/2}}$$

- In zero gravity ($\mathrm{Bo} = 0$, top=bottom):

$$x'(z') = \frac{1}{\cos\theta_c} \left\{1 - \left[1 - (1 - z')^2 \cos^2\theta_c\right]^{1/2}\right\}$$

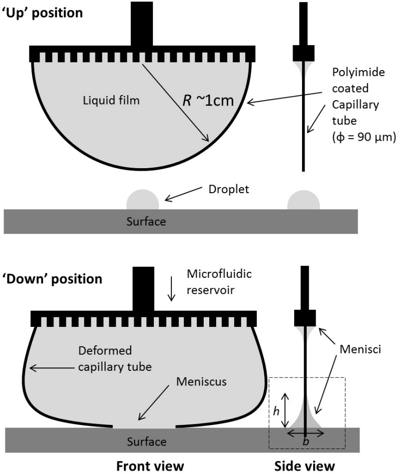
Simulation: Surface Evolver (SE)

- Discretise each interface and perform direct numerical minimisation of surface energy for a fixed PB area.
- Only half PB is simulated, by symmetry.
- Discretisation induces a small, unphysical, 'contact' angle where the PBs meet the vertical film.

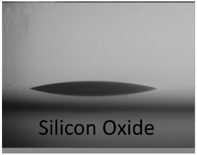


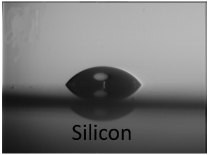
Experiment: set-up

- Contact angle meter (GBX Scientific Instruments, France).
- Commercially available soap solution (Pustefix, Germany), surface tension $\gamma = 28.2 \pm 0.3$ mJ m^{-2}, $\lambda_c \approx 1.7$ mm.
- In-house microfluidic tool consisting of (i) microfluidic reservoir with a number of capillary slots, made of ABS plastic; (ii) thin, flexible, hydrophilic loop which supports liquid film, made of polyimide-coated capillary tubing (Molex, USA). **Gives bottom PB shape only, not top.**
- We measure: meniscus width at substrate $2x$; meniscus height h.

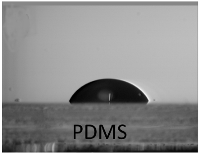


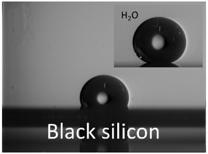
Experiment: substrates studied

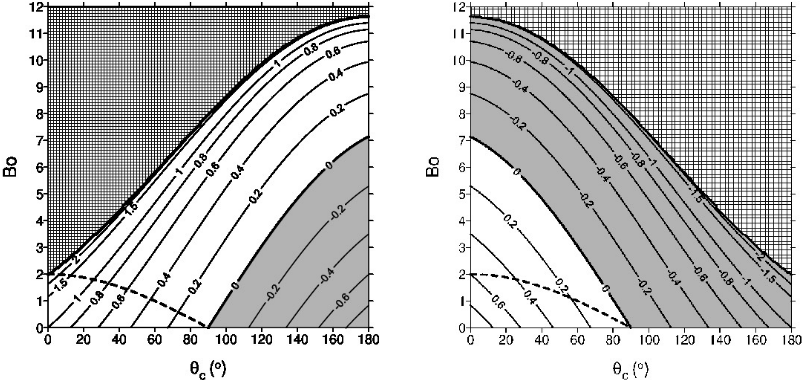




Material	θ_c (°)
SiO_2	18.2 ± 2.8
Teflonised polished Si	51.7 ± 0.3
PDMS elastomer	61.0 ± 2.1
Teflonised rough Si	64.0 ± 0.4
Teflonised black Si	109.3 ± 0.3

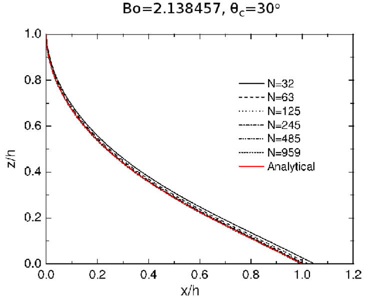


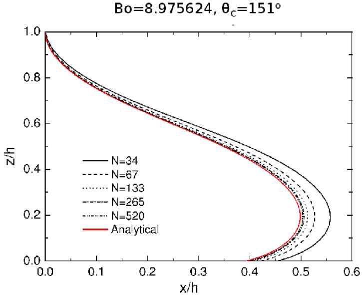




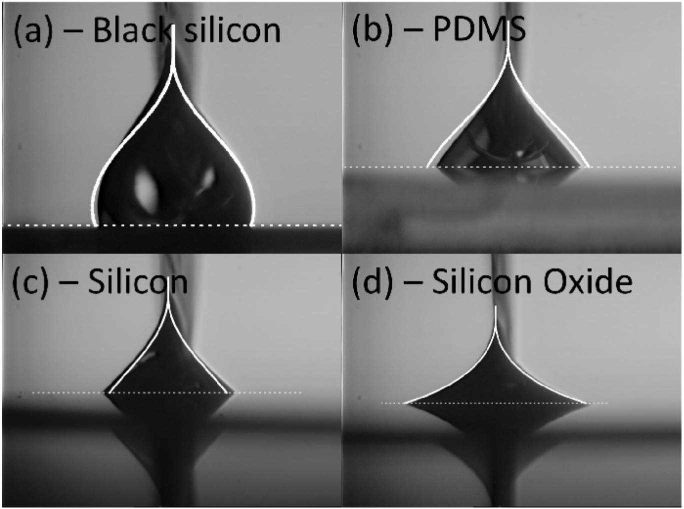
Domains of allowed (white, below dashed line only in right panel) and forbidden (elsewhere) PBs in (θ_c, Bo) space, at the bottom (left) and top (right) substrates. The solid lines are loci of constant $x'(z'=0)$. The dashed line is $\mathrm{Bo} = 2\cos\theta_c$.

Results: bottom PB shapes, theory + SE



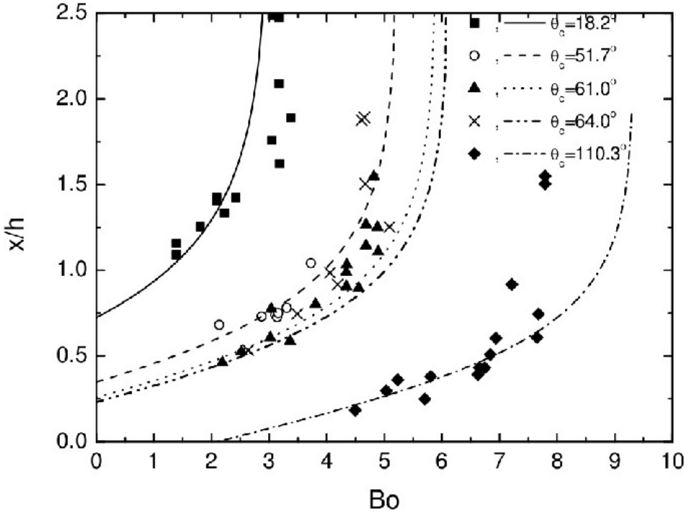


Comparison of bottom PB shapes from analytical theory and Surface Evolver.



PBs at four of the five surfaces used in the experiments.

Results: bottom PB shapes, theory + experiment II

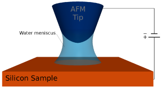


Scaled PB half-width $x(z=0)/h$ *vs* Bond number, from theory (curves) and experiment (symbols).

Liquid bridges

- If the film connecting the two PBs is not infinitesimally thin we have a liquid bridge or capillary bridge. May cause repulsion or attraction.
- Liquid bridges are relevant in many contexts:
  - Sand art

  - AFM in high-humidity environments



  - Soldering

  - Lungs, closing small airways and impairing gas exchange
  - Wet adhesion of insects and tree frogs

Theory: the physics of liquid bridge shape

- Bridge shape is determined by the balance of gravity, surface tension, and liquid contact angles. So a key dimensionless quantity is the Bond number (H is substrate separation):

$$\mathrm{Bo} = \frac{\rho g H^2}{\gamma}$$

- Relevant lengthscale is the capillary length:

$$\lambda_c = \left(\frac{\gamma}{\rho g} \right)^{1/2}$$

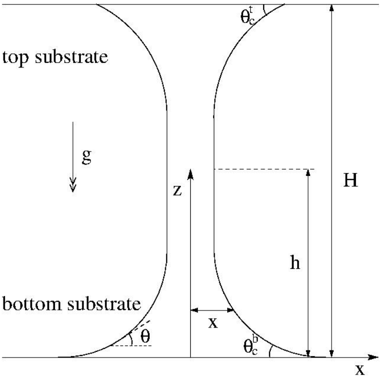
- Starting with Lagrange, a lot of work has been done on axisymmetric bridges.
- We solve the Young-Laplace equation with gravity for a planar bridge between two horizontal, flat substrates, to predict the bridge shape.

Theory: liquid bridge and the Young-Laplace equation

Young-Laplace equation for liquid bridge between two planar substrates:

$$\left[1+\left(\frac{dx}{dz}\right)^2\right]^{-3/2}\frac{d^2x}{dz^2}=-\frac{\Delta p}{\gamma}$$

$\Delta p = [p_b(z=0)-\rho g z]-p_a$ is pressure difference across the interface ($\propto$ curvature), γ is liquid-gas surface tension.



Boundary conditions:

1. $dx/dz(z=0)=-\cot\theta_c^b$ (bottom substrate);
2. $dz/dx(x=H)=\cot\theta_c^t$ (top substrate).

Theory: solving the Young-Laplace equation

- Rewrite equation in terms of film inclination $\theta(z)$: boundary conditions are $\theta(0) = \theta_c^b$, $\theta(H) = \theta_c^t$:

$$\sin\theta \, \frac{d\theta}{dz} = -\frac{\Delta p}{\gamma}$$

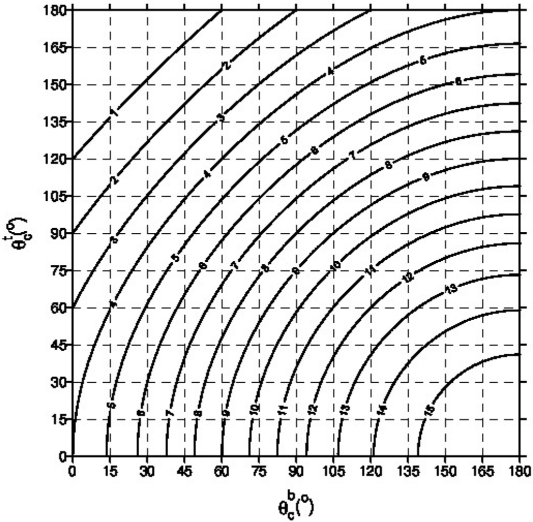
- Analytically exact solution (lenghts in units of H):

$$x'(z') = x'(0) - \int_0^{z'} \frac{-\cos\theta_c^t z'' + (1-z'')\left(\cos\theta_c^b + \frac{\mathrm{Bo}}{2} z''\right)}{\left\{1 - \left[-\cos\theta_c^t z'' + (1-z'')\left(\cos\theta_c^b + \frac{\mathrm{Bo}}{2} z''\right)\right]^2\right\}^{1/2}} dz''$$

- In zero gravity ($\mathrm{Bo} = 0$, top=bottom):

$$x'(z') = x'(0) + \frac{\sin\theta_c^b - \left\{1 - \left[\cos\theta_c^b - \left(\cos\theta_c^b + \cos\theta_c^t\right) z'\right]^2\right\}^{1/2}}{\cos\theta_c^b + \cos\theta_c^t}$$

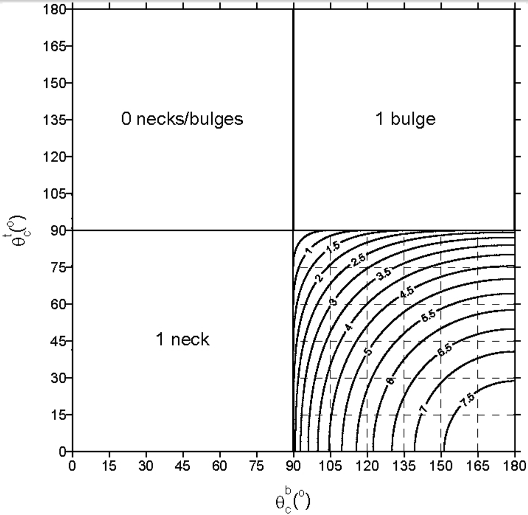
Results: when does a bridge bridge?



From requiring that $-1 \leq \cos\theta(z) \leq 1$:

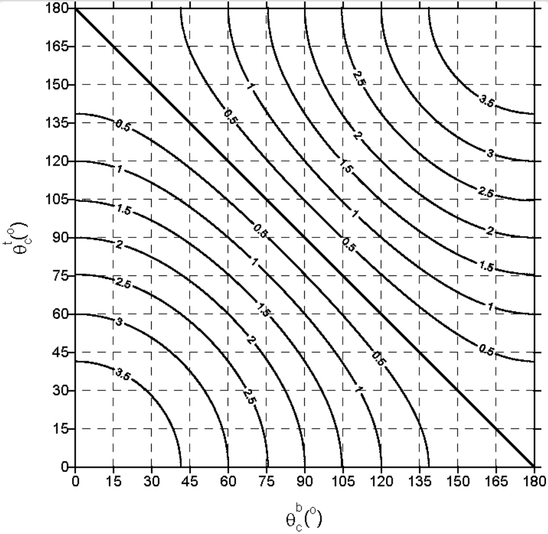
$$\mathrm{Bo} \leq \mathrm{Bo}^{max} = 2\left(2 - \cos\theta_c^b + \cos\theta_c^t\right)) + 4\sqrt{\left(1 - \cos\theta_c^b\right)\left(1 + \cos\theta_c^t\right)}$$

Results: how many necks and bulges?



From requiring that $\theta(z) = 90^\circ$ (in units of H):

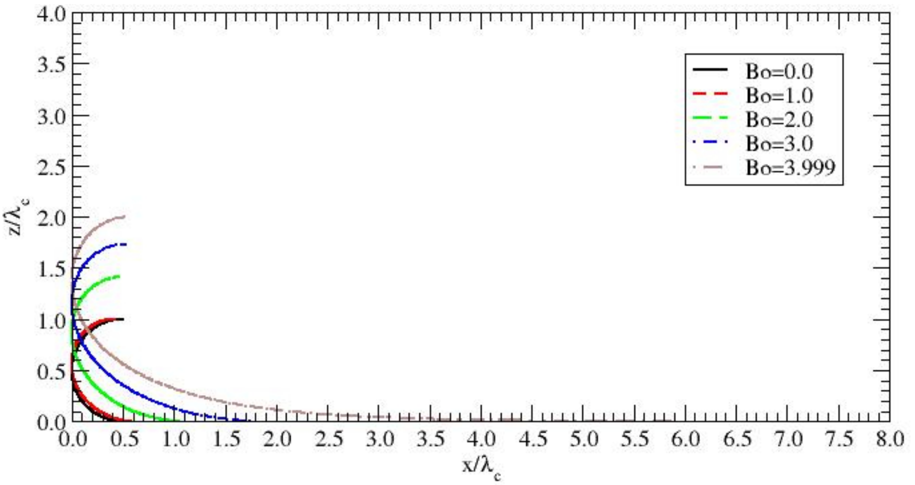
$$h' = \frac{\mathrm{Bo} - 2\left(\cos\theta_c^b + \cos\theta_c^t\right) \pm \sqrt{\mathrm{Bo}^2 - 4\left(\cos\theta_c^t - \cos\theta_c^b\right)\mathrm{Bo} + 4\left(\cos\theta_c^t + \cos\theta_c^b\right)^2}}{2\mathrm{Bo}}$$



From requiring that $d\theta(z)/dz = 0$ (in units of H):

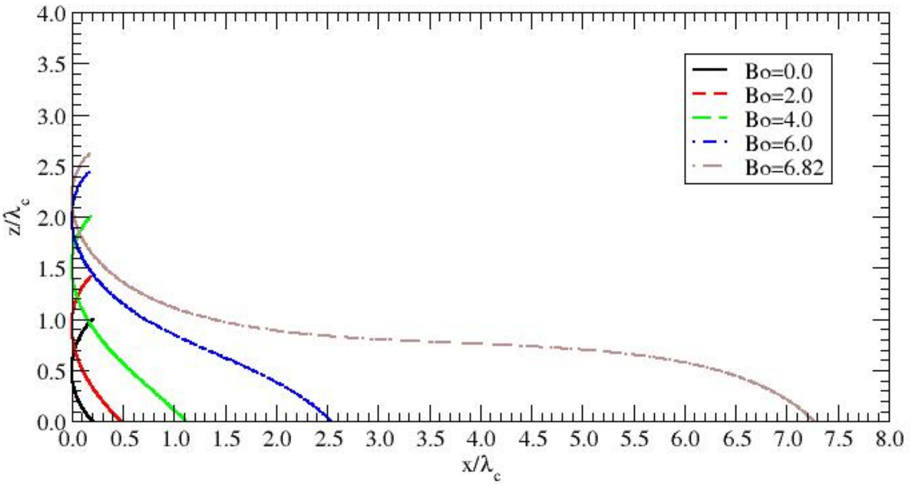
$$\tilde{h}' = \frac{1}{2} - \frac{\cos\theta_c^b + \cos\theta_c^t}{\mathrm{Bo}}$$

Results: bridge shapes *vs* Bo, $\theta_c^b = \theta_c^t = 0°$



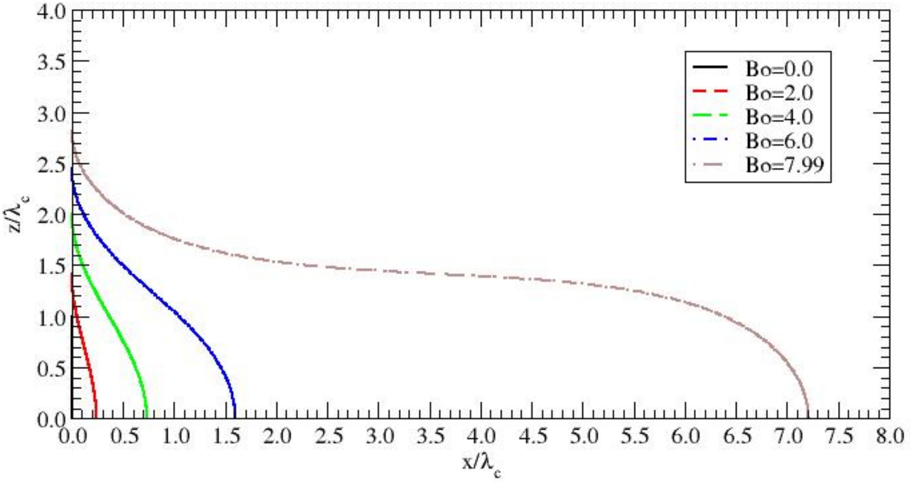
Lengths scaled by capillary length λ_c.

Results: bridge shapes *vs* Bo, $\theta_c^b = \theta_c^t = 45^\circ$



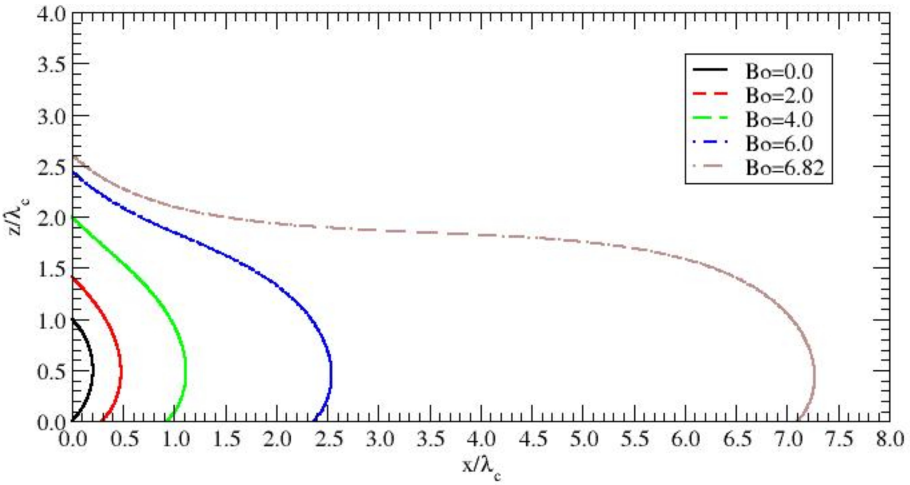
Lengths scaled by capillary length λ_c.

Results: bridge shapes *vs* Bo, $\theta_c^b = \theta_c^t = 90^\circ$



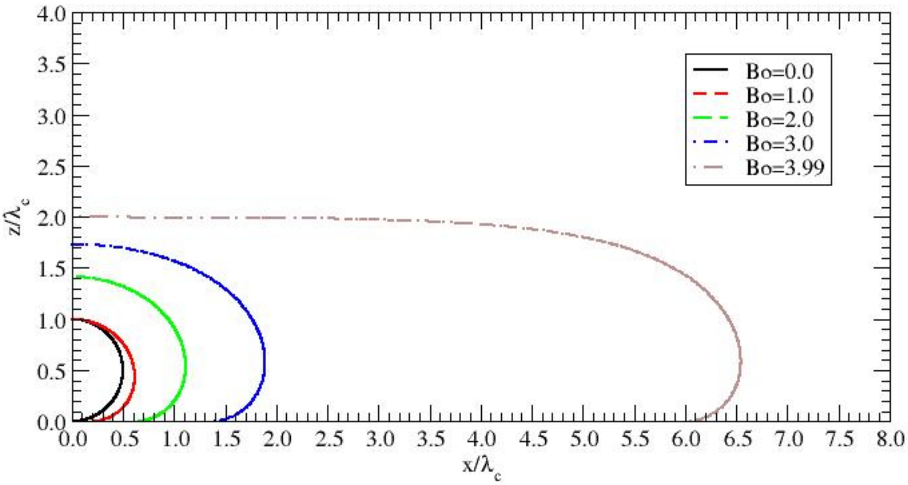
Lengths scaled by capillary length λ_c.

Results: bridge shapes *vs* Bo, $\theta_c^b = \theta_c^t = 135^\circ$



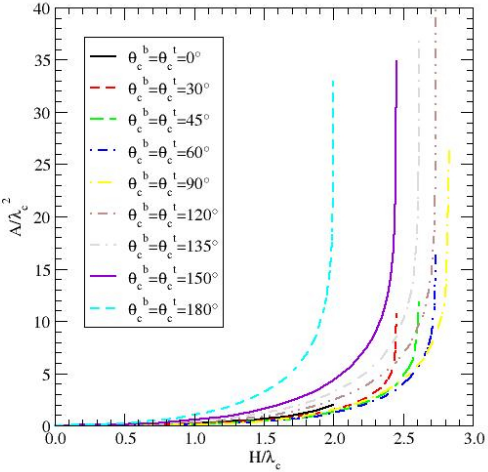
Lengths scaled by capillary length λ_c.

Results: bridge shapes *vs* Bo, $\theta_c^b = \theta_c^t = 180°$



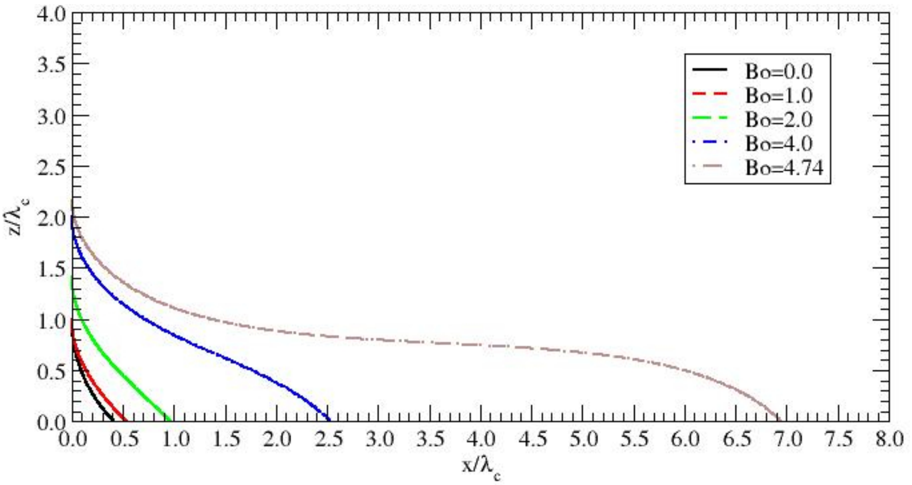
Lengths scaled by capillary length λ_c.

Results: minimum X-sectional area, $\theta_c^b = \theta_c^t$



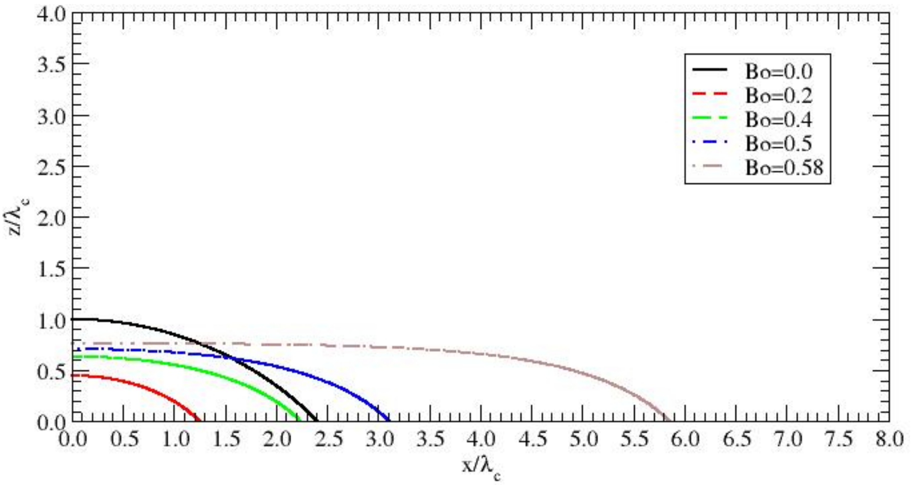
Lengths scaled by capillary length λ_c.

Results: bridge shapes *vs* Bo, $\theta_c^b = 45^\circ$, $\theta_c^t = 90^\circ$



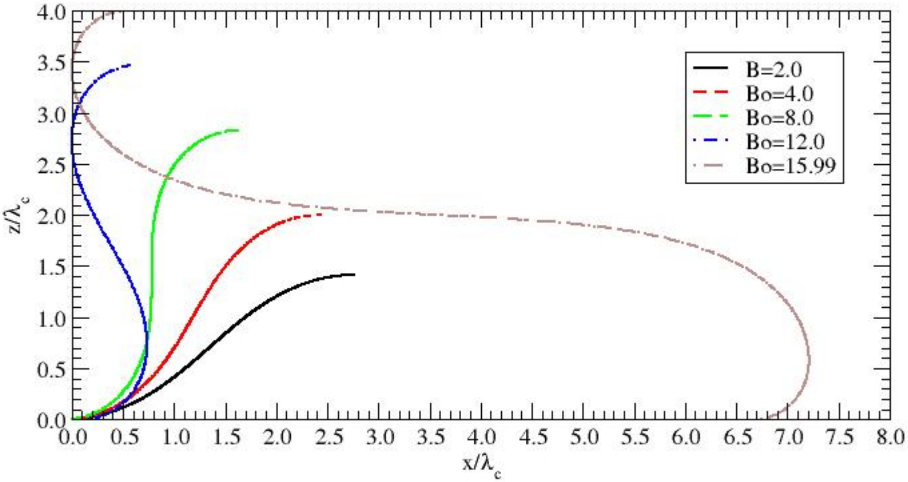
Lengths scaled by capillary length λ_c.

Results: bridge shapes *vs* Bo, $\theta_c^b = 45°$, $\theta_c^t = 180°$



Lengths scaled by capillary length λ_c.

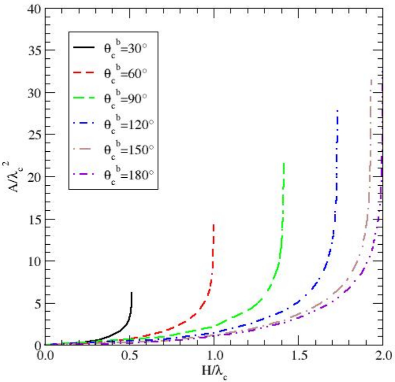
Results: bridge shapes *vs* Bo, $\theta_c^b = 180^\circ$, $\theta_c^t = 0^\circ$



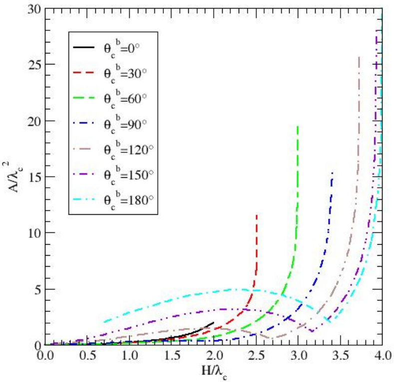
Lengths scaled by capillary length λ_c.

Results: minimum X-sectional area, $\theta_c^b \neq \theta_c^t$

$\theta_c^b \leq \theta_c^t = 180°$



$\theta_c^b \geq \theta_c^t = 0°$



Lengths scaled by capillary length λ_c.

Summary and conclusions

- We have integrated the Young-Laplace equation (quasi-)analytically to find the shape of the 2D PBs along which a planar vertical film meets two horizontal flat substrates of given wetttabilities.
- The combination of a particular surface (θ_c) in contact with a particular foam (ρ and γ) leads to allowed and forbidden surface PBs: a surface can be **foam-philic** or **foam-phobic**.
  - PBs at the top substrate can only exist in a small region of (θ_c, Bo) space; in particular, one must have $\theta_c < 90^\circ$.
  - PBs at the bottom substrate a have wider range of existence, requiring larger θ_c at higher Bo or vice versa.
- Predictions are in fairly good agreement with experiment.
- This was then generalised to a 2D liquid bridge: we established the range of substrate separations for which the bridge can exist, as well as the positions of any necks/bulges amd inflection points on its surface. These results are analytically exact.
- We also obtained the minimum cross-sectional area of such a liquid bridge, as a function of contact angles and substrate separation.

Acknowledgements

We acknowledge funding from the following sources:

- French RENATECH network.
- European Commission through MSCA-RISE project Matrixassay (ID: 644175).
- Fundação para a Ciência e Tecnologia (Portugal) through grants EXCL/FIS-NAN/0083/2012 and UID/FIS/00618/2013.
- Instituto Politécnico de Lisboa (IPL) for travel support.
- Wilhelm und Else Heraeus Stiftung for hospitality.

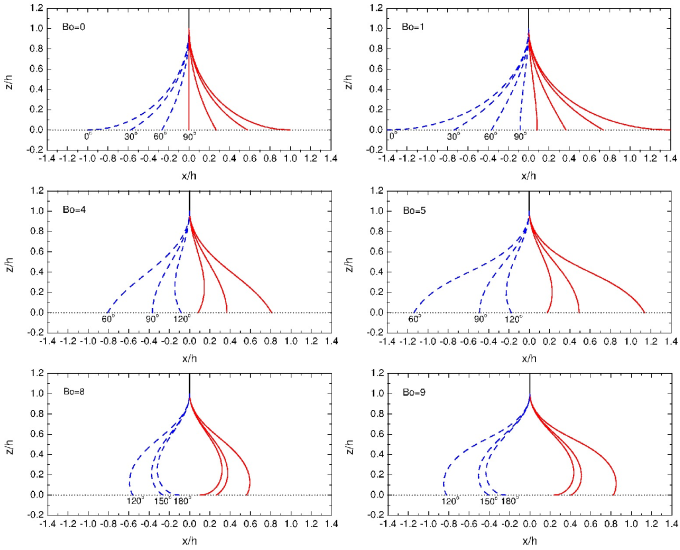
and would like to thank K. Brakke for his development and maintenance of the Surface Evolver code.

Shameless publicity plug. . .

Topics

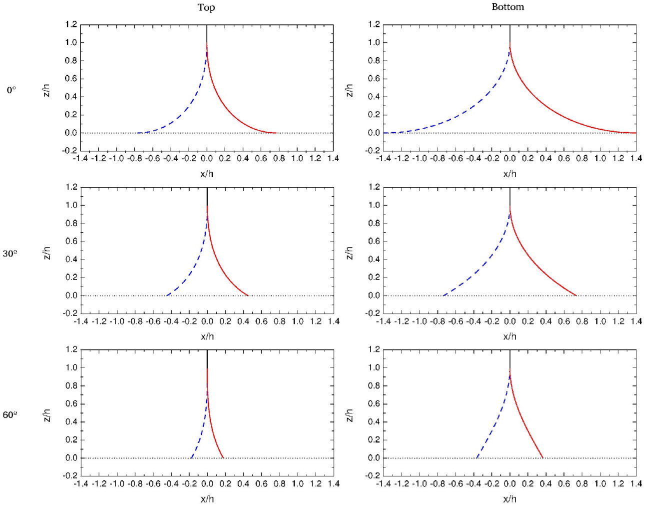
- Liquid Crystals in Biology and Active Matter
- Macromolecular Liquid Crystals
- Confined Liquid Crystals
- Design of New Materials
- Mathematical Modelling, Symmetry and Topology
- Novel Applications

Results: bottom PB shapes, theory



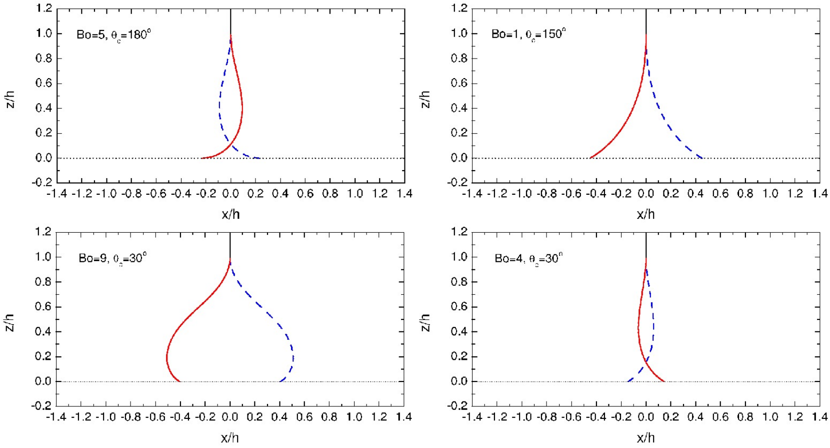
Analytically-calculated PB shapes at bottom substrate, for various combinations of Bo and θ_c.

Results: top and bottom PB shapes, theory

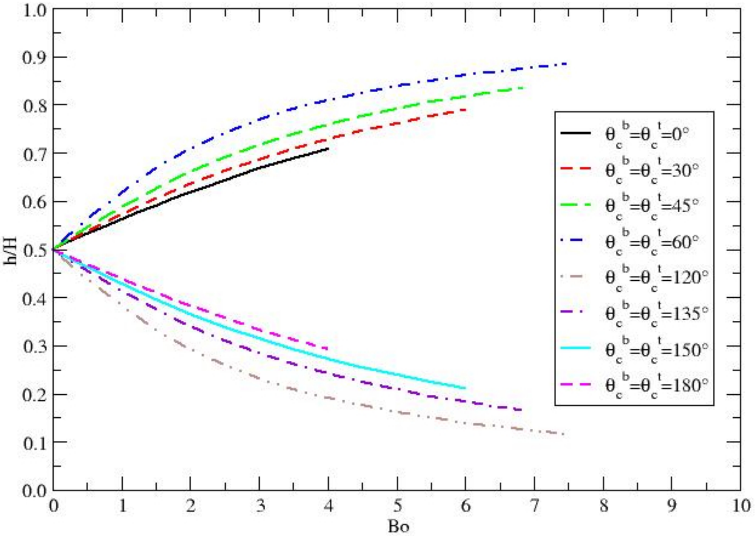


Analytically-calculated PB shapes at top (left column) and bottom (right column) substrates, for $\mathrm{Bo} = 1$ and $\theta_c = 0^\circ$ (top row), 30° (centre row) and 60° (bottom row).

Results: unphysical PB shapes, theory

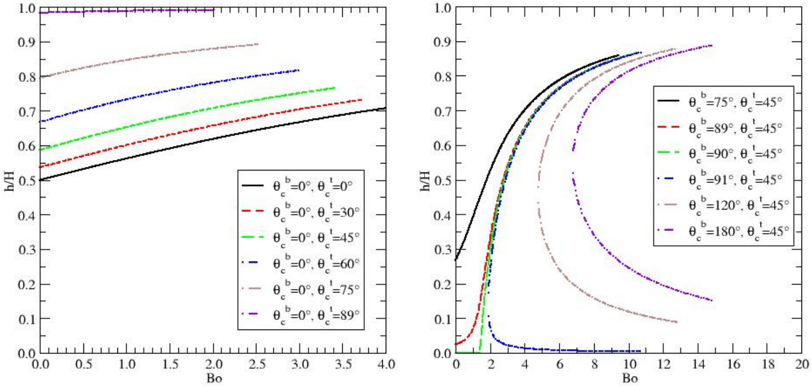


Examples of unphysical bottom PBs (top row) and top PBs (bottom row).



Lengths scaled by substrate separation *H*.

Results: position of neck/bulge *vs* Bo, $\theta_c^b \neq \theta_c^t$



Lengths scaled by substrate separation H.